Modeling System Dynamics in a MEMS-Based Stirling Cooler

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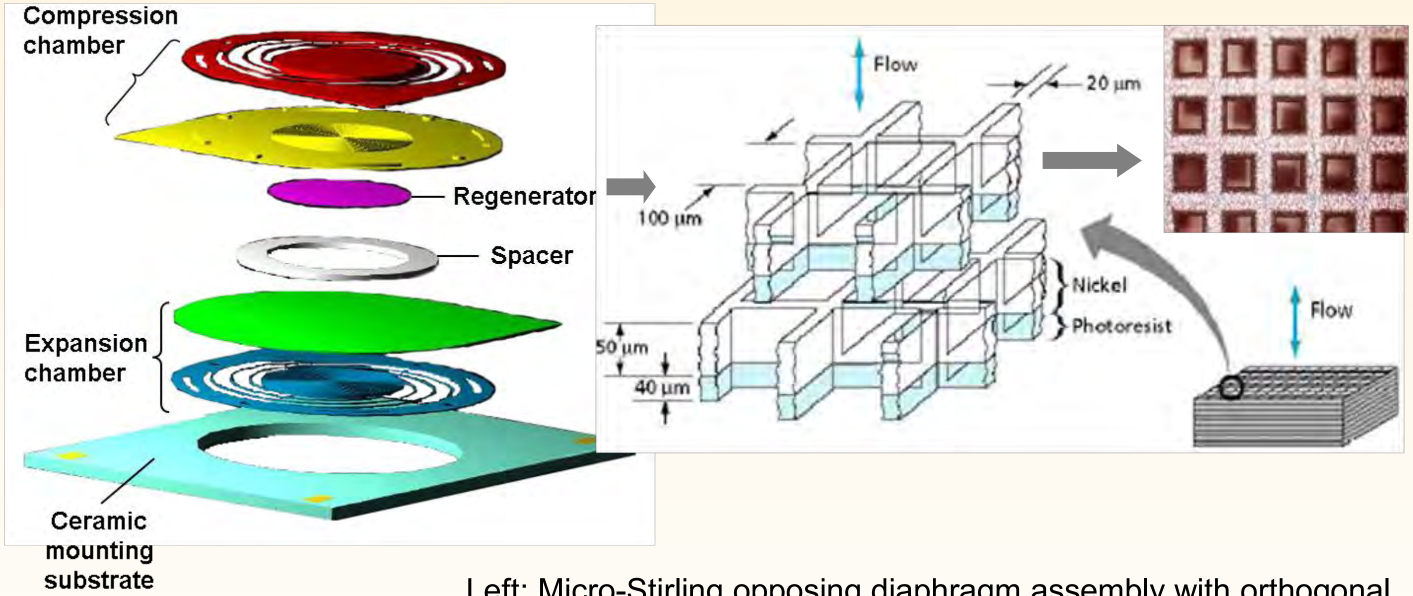
Motivation

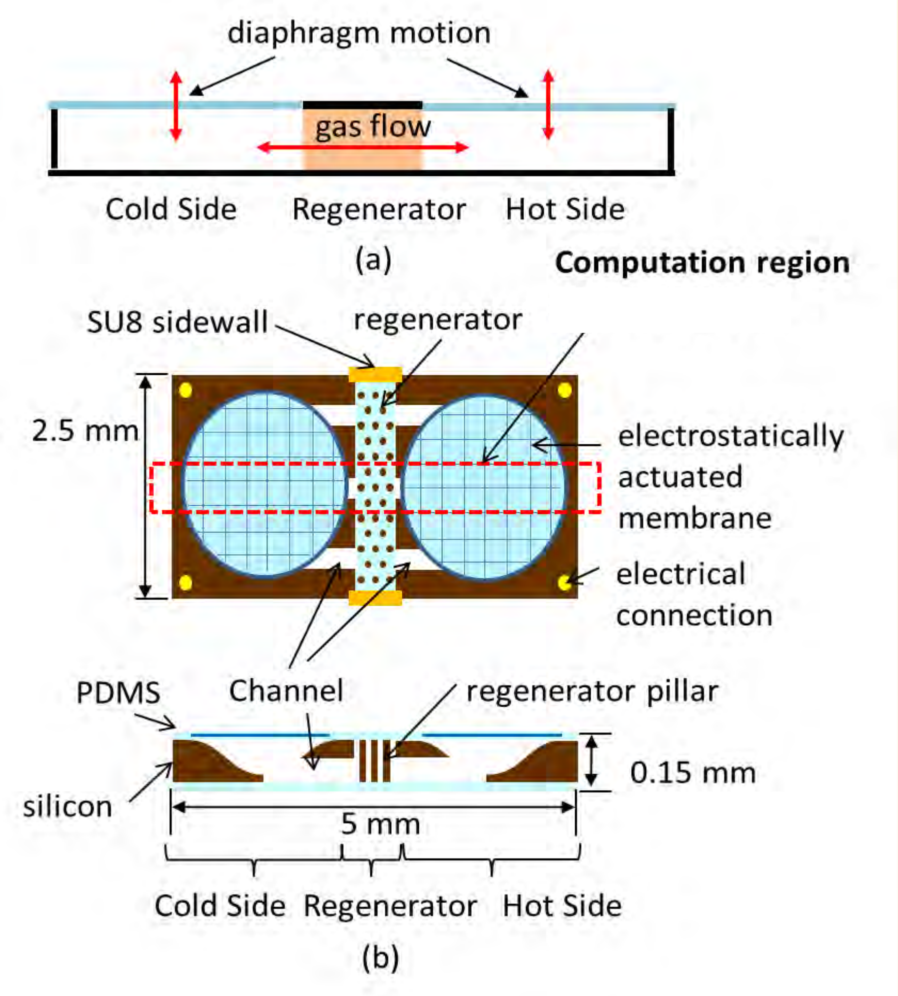
- Application areas: Cooling for chip- and board-level electronics, sensors and radio frequency systems.
- Thermoelectric coolers: Significant challenges exist for a high efficiency of thermoelectric energy conversion.



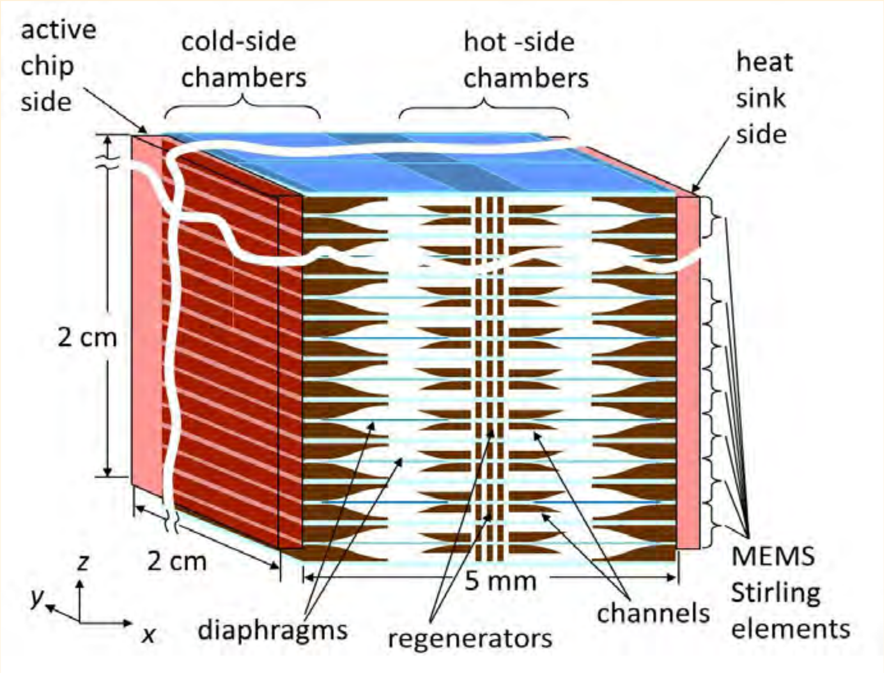
Left: Micro-Stirling opposing diaphragm assembly with orthogonal gas flow. Right: Nickel-photoresist regenerator design showing orthogonal flow between opposed diaphragms.

Ref: M.Moran et al., IECEC 2004

Stirling Cooler Design



(a) Conceptual schematic of a single element of the Stirling micro-cooler. (b) Top: plan view; and Bottom: cross-sectional view of the element.

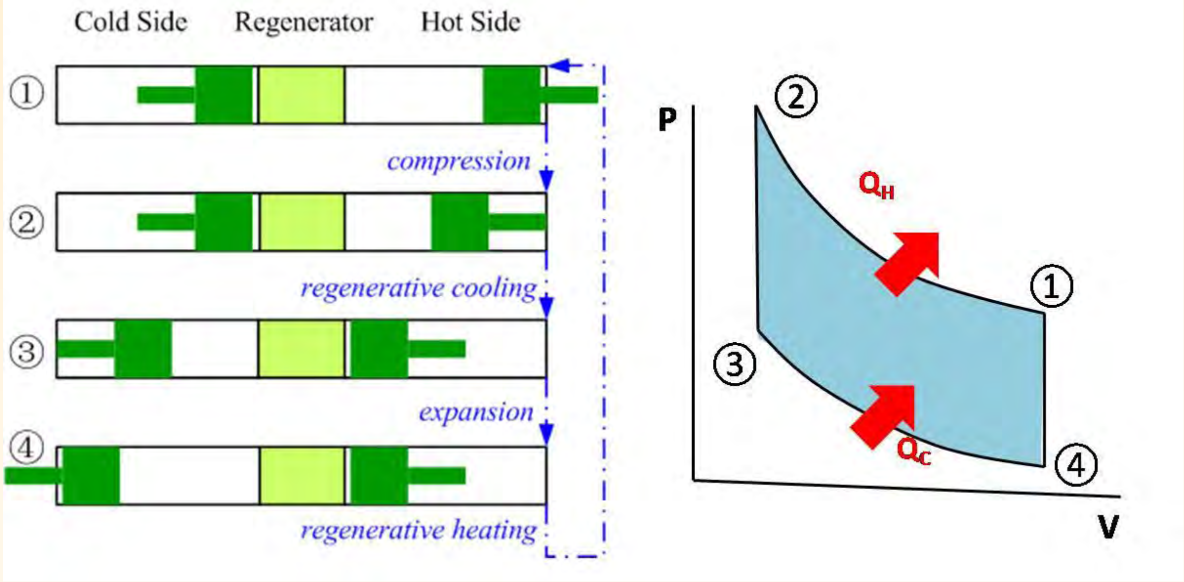


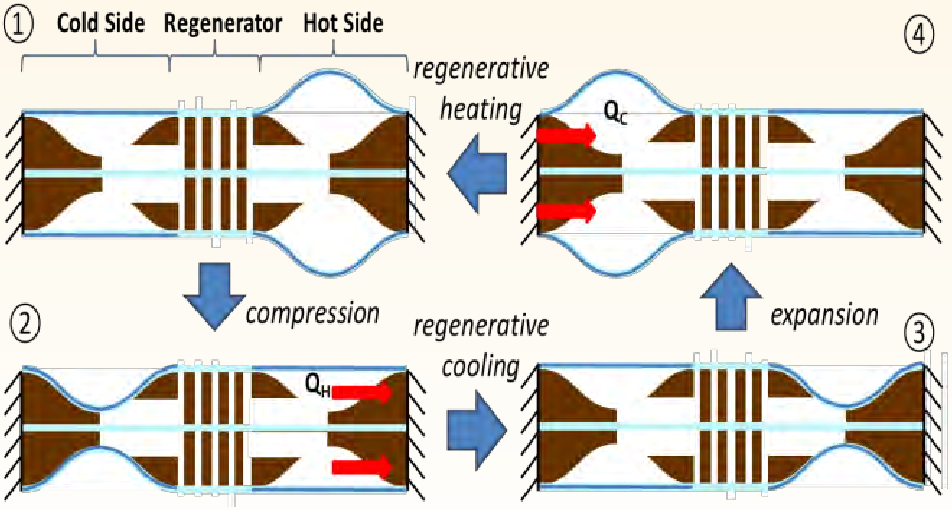
Stirling Cooler Element

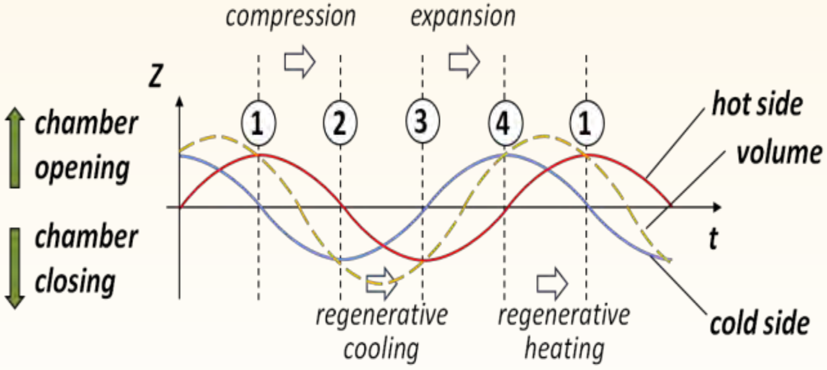
- 5 mm × 2.5 mm × 0.15 mm
- Two diaphragm actuators and a regenerator
- Driven electrostatically
- Working fluid: Air
- Stacked in parallel

Stirling Cycle

- ①-②: Isothermal compression
- ②-③: Constant-volume regenerative cooling
- ③-④: Isothermal expansion
- ④-①: Constant-volume regenerative heating







Governing Equations

- Compressible flow and heat transfer

Continuity equation

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0}$$

Momentum equation

$$\boxed{\rho \frac{\partial \vec{u}}{\partial t} + \rho(\vec{u} \cdot \nabla)\vec{u} = -\nabla \mathrm{p} - \nabla \cdot \bar{\bar{\tau}}}$$

Energy equation

$$\boxed{\rho C_p \left[\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla)\mathrm{T}\right] = \nabla \cdot (k \nabla T) + \bar{\bar{\tau}} : \nabla \vec{u} - \frac{T}{\rho}\frac{\partial \rho}{\partial T}\left(\frac{\partial p}{\partial t} + \vec{u} \cdot \nabla p\right)}$$

Viscous tensor

$$\boxed{\bar{\bar{\tau}} = \mu[\nabla \vec{u} + (\nabla \vec{u})^T] - \frac{2}{3}\mu(\nabla \cdot \vec{u})}$$

Ideal gas

$$\boxed{\rho = \frac{p M_g}{RT}}$$

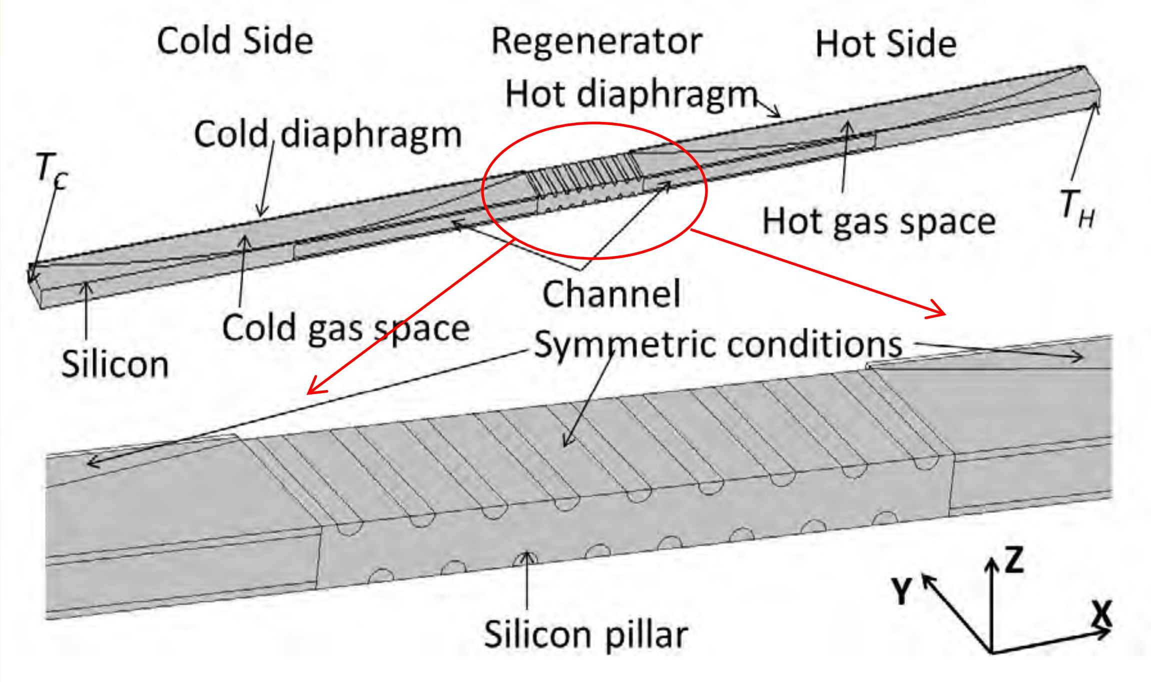
- Heat transfer in solid

$$\boxed{\rho_s C_{p_s} \frac{\partial T}{\partial t} = \nabla \cdot (k_s \nabla T)}$$

Setup & Boundary Conditions

Physics

- Non-isothermal Flow module
- The ALE (Arbitrary Lagrangian-Eulerian) moving mesh method
- Solid Mechanics module for moving diaphragms.

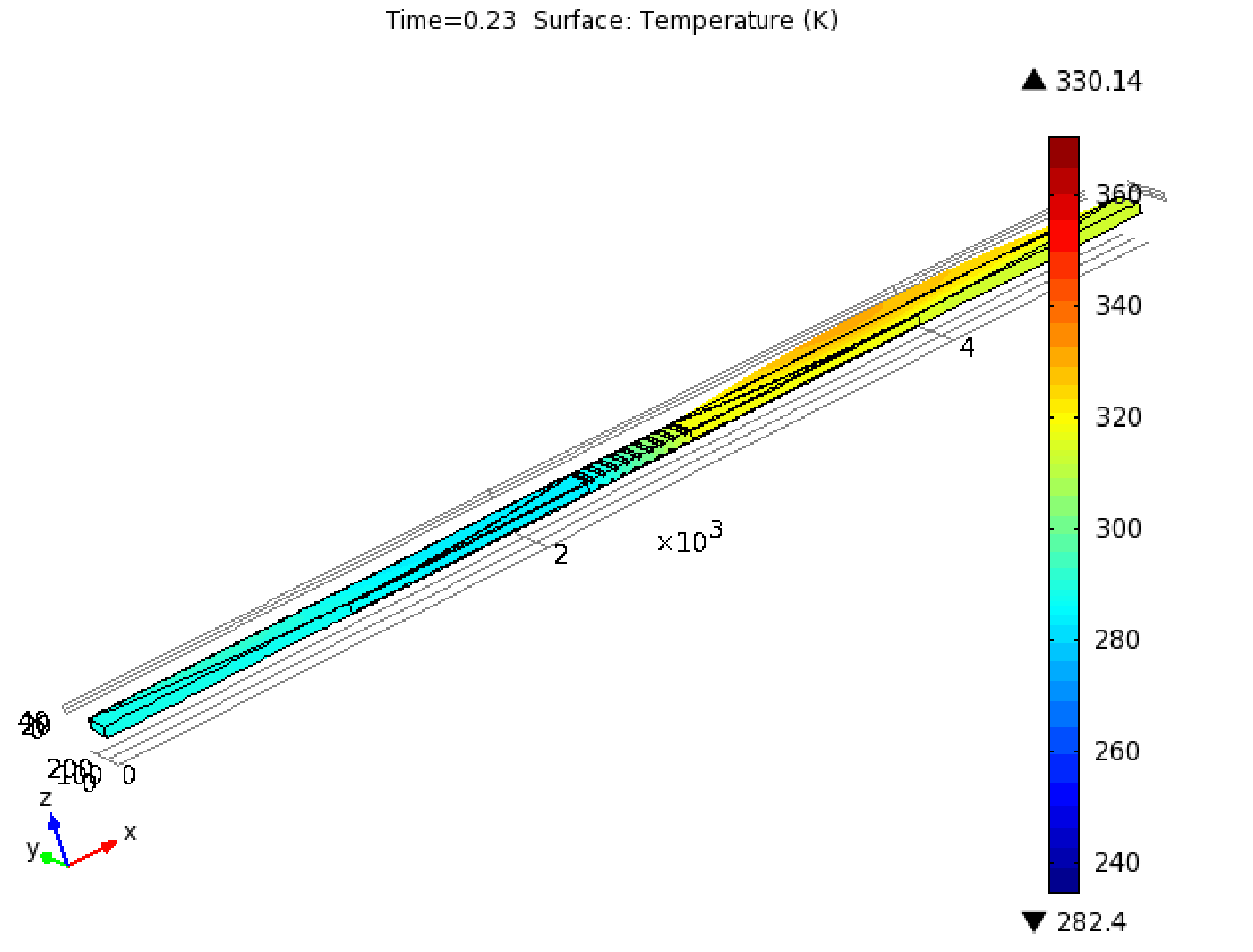


Setup

- Sinusoidal motions, 90° out of phase for the cold and hot diaphragms
- T_C= 288.15 K &T_H = 313.15 K
- Operating frequency 100 Hz
- Initial pressure 2 bar
- Circular silicon pillars with 20 µm diameter

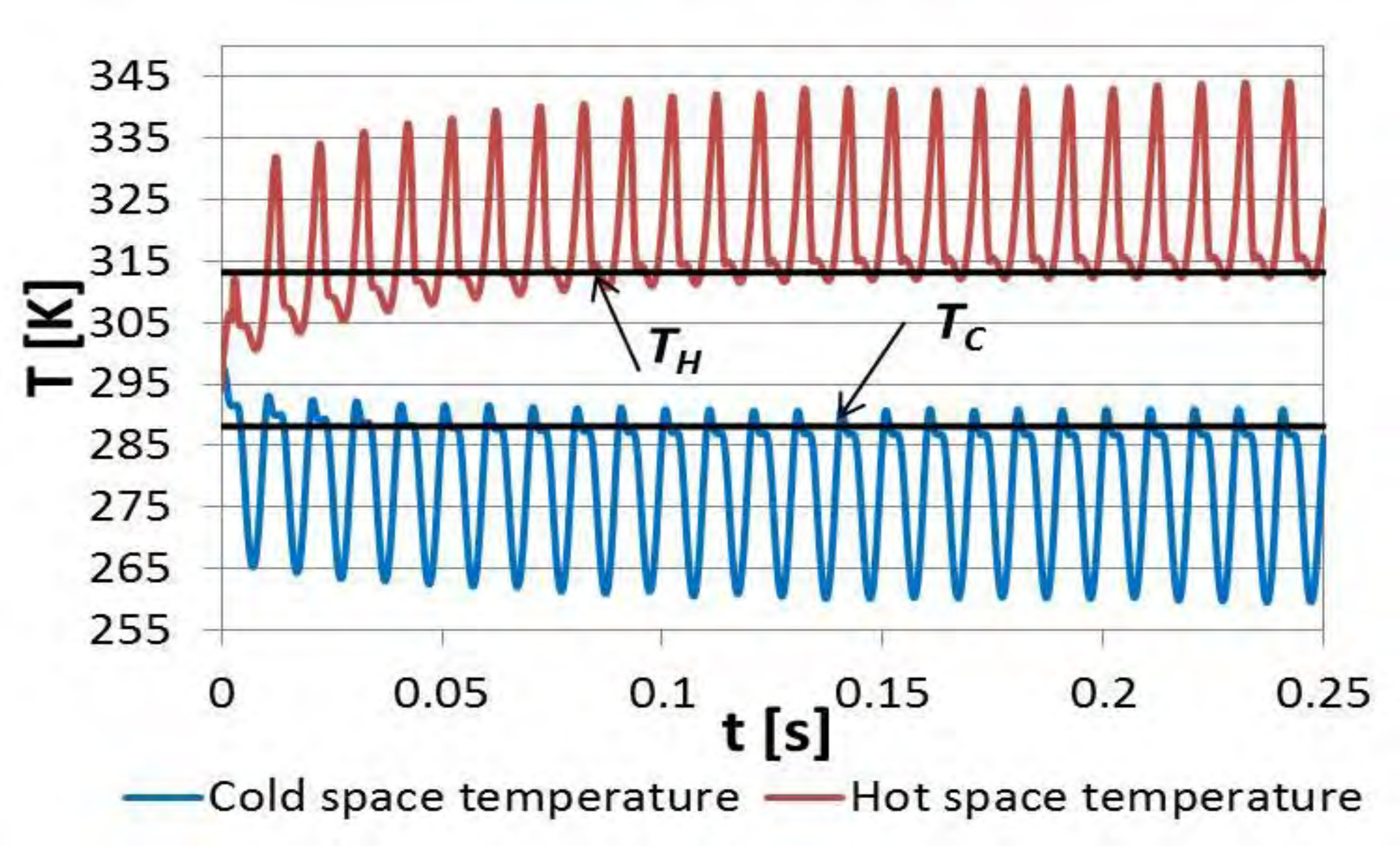
Results and Discussions

Temperature contour



Results and Discussions

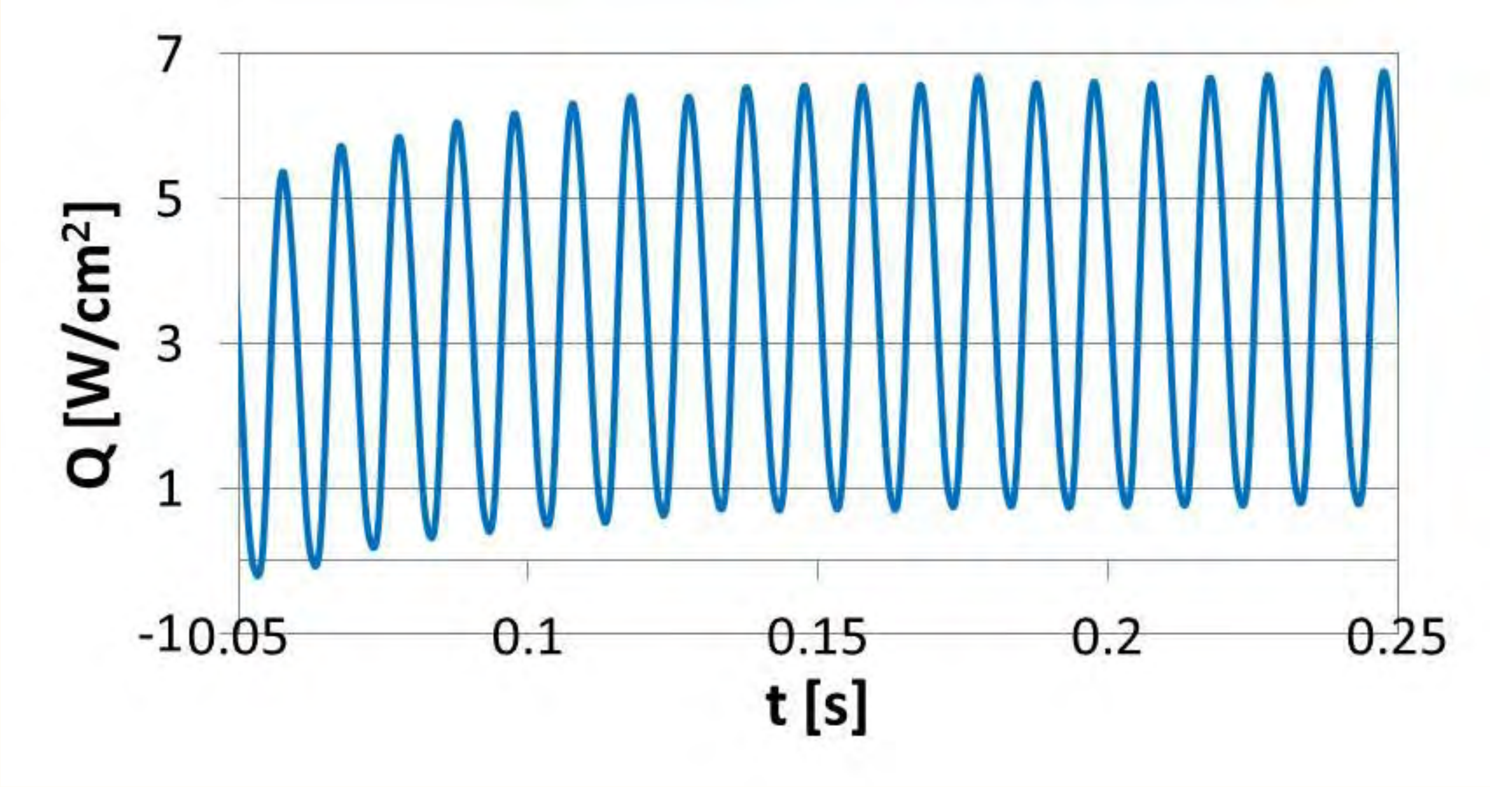
Space-averaged temperature



- Cold side space-averaged temperature is 277.5 K; T_C= 288.15 K
- Hot side space averaged temperature is 322.5 K; T_H = 313.15 K

Results and Discussions

Averaged heat flux coming into the cold chamber



- Average cooling capacity of the Stirling cooler element is 3.74 W/cm^2

Results and Discussions

COP (Coefficient Of Performance)

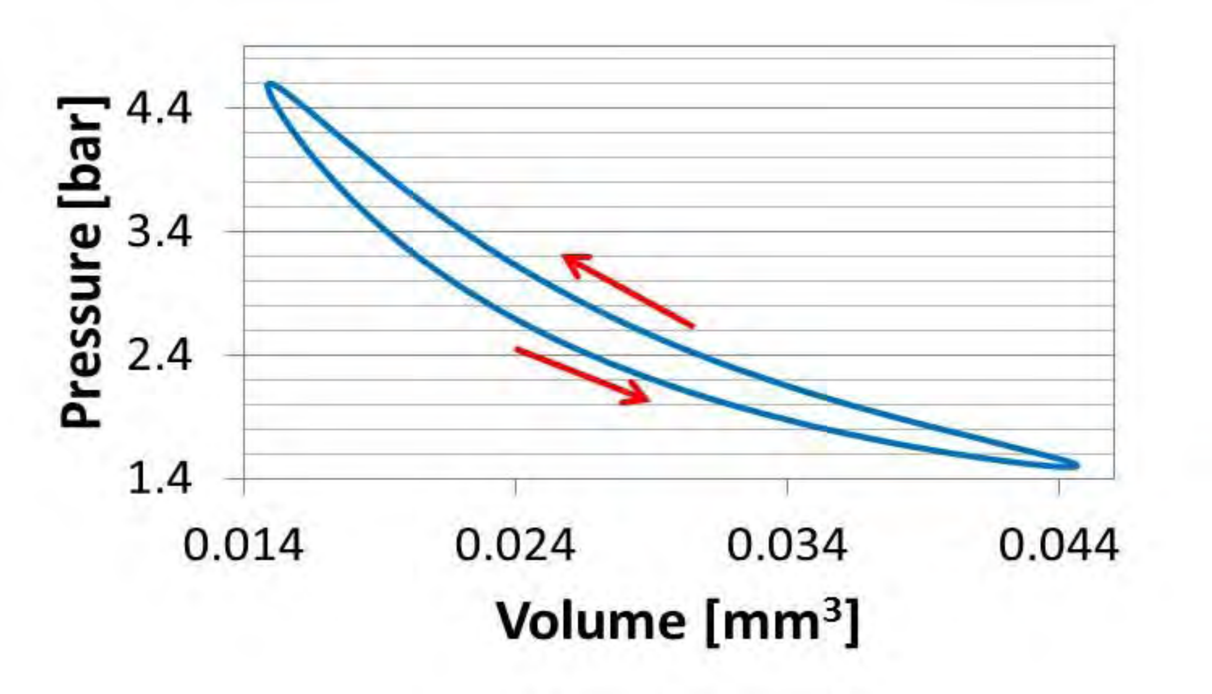
- *COP* of a cooler is the ratio of the heat removal to the input work, $COP = \frac{q}{W}$
- The Carnot *COP*

$$COP_C = \frac{T_C}{T_H - T_C} = 11.5$$

- *COP* calculated is

$$COP = 5.6$$

- $\frac{COP}{COP_C} = 48.7\%$



P-V cycle of the system

Conclusions and Future Work

- COMSOL Multiphysics program has been used to simulate of a new Stirling micro-scale cooler.
- The model results predicted the cooling capacity and *COP* of the cooler.
- Future work will focus on the performance of the whole system device and the regenerator will be replaced by porous media for simplifying the full system model.
- The system model coupling with electrostatics is to be studied for obtaining more accurate motions of the diaphragms.

THANK YOU!