Magnetic Manipulation of Lateral Migration Behavior of a Ferrofluid Droplet in a Plane Poiseuille Flow

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INTRODUCTION: A detailed investigation on the effect of a uniform magnetic field on the lateral migration of a ferrofluid droplet in a plane Poiseuille flow by means of numerical simulation is presented here. In this case, the magnetic field is applied along different arbitrary directions.

COMPUTATIONAL METHODS: The conservative level set method is used to track the dynamic interface of the droplet where the level set function ϕ is advected by the velocity field[1,2]:

$$\frac{d\phi}{dt} + \mathbf{u}\cdot\nabla\phi = \gamma\nabla\cdot\left(\varepsilon\nabla\phi - \phi(1-\phi)\frac{\nabla\phi}{|\nabla\phi|}\right)$$

Being treated as a single phase flow, the different properties of the flow domain are related to ϕ through the following equations:

$$\rho = \rho_c + (\rho_d - \rho_c)\phi;\;\; \eta = \eta_c + (\eta_d - \eta_c)\phi$$

$$\mu = \mu_c + (\mu_d - \mu_c)\phi;\;\; \chi = \chi_c + (\chi_d - \chi_c)\phi$$

The flow field under the effect of uniform magnetic field can be governed by the continuity and momentum equations:

$$\nabla\cdot\boldsymbol{u} = 0$$

$$\rho\left(\frac{\delta\mathbf{u}}{\delta t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\boldsymbol{\tau} + \mathbf{F}_\sigma + \mathbf{F}_m$$

where, the surface tension force, $\mathbf{F}_\sigma$ can be defined as:

$$\mathbf{F}_\sigma = \nabla\cdot[\sigma\{\mathbf{I} + (-\mathbf{n}\mathbf{n}^T)\}\delta]$$

and magnetic force, $\mathbf{F}_m$ can be calculated as:

$$\mathbf{F}_m = \nabla\cdot\boldsymbol{\tau}_m = \nabla\cdot\left(\mu\mathbf{H}\mathbf{H}^T - \frac{\mu}{2}H^2\mathbf{I}\right)$$

The magneto-static Maxwell equation can be written as:

$$\nabla\cdot\mathbf{B} = 0,\;\; \nabla\times\mathbf{H} = 0,\;\; \nabla\cdot(\mu\nabla\varphi) = 0$$

$$\mathbf{M} = \chi\mathbf{H} \;\text{ and }\; \mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mu_0(1+\chi)\mathbf{H}$$

Dimensionless Groups:

- ❑ $Re_d = \frac{\rho_c R_0^2 \dot{\gamma}_a}{\eta_c}$
- ❑ $\mathrm{Bo_m} = \frac{R_0 \mu_0 H_0^2}{2\sigma}$

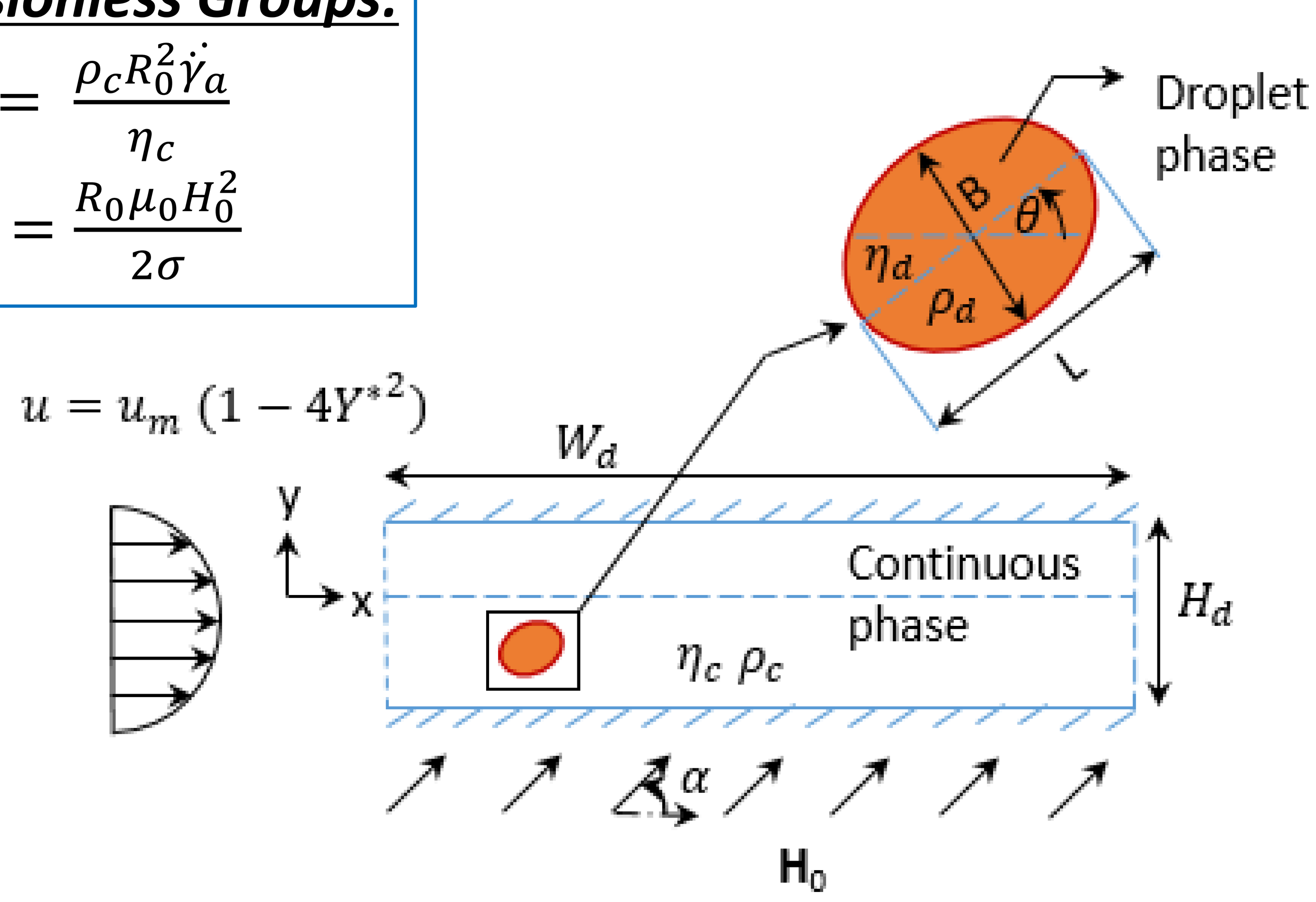

Figure 1. Schematic illustration of a ferrofluid droplet suspended in another medium in a Poiseuille flow under the application of a uniform magnetic field, $\mathbf{H}_0$.

RESULTS: The effect of different arbitrary magnetic field directions on the final equilibrium position of the droplet

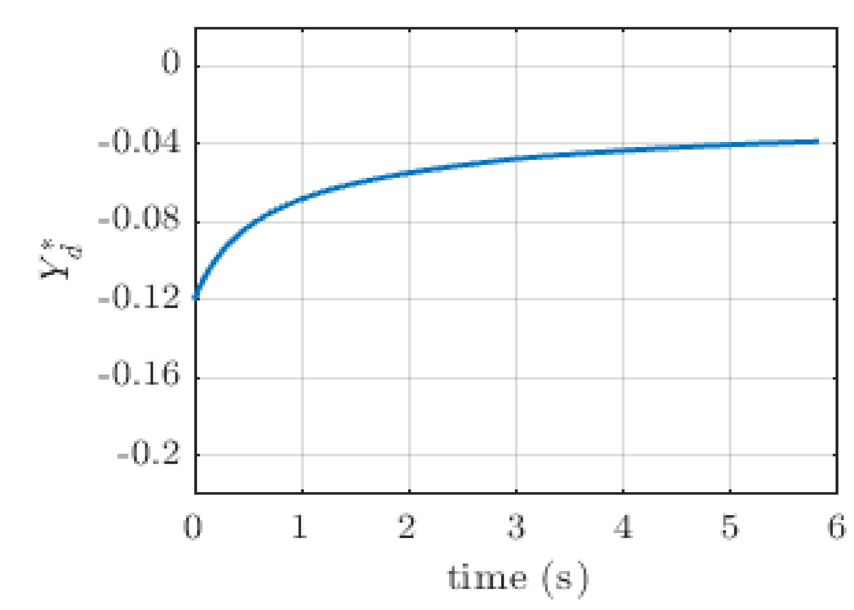

Figure 2. Lateral migration of a ferrofluid droplet at $Re_d = 0.03$, $\lambda = 1$, and $Bo_m = 0$.

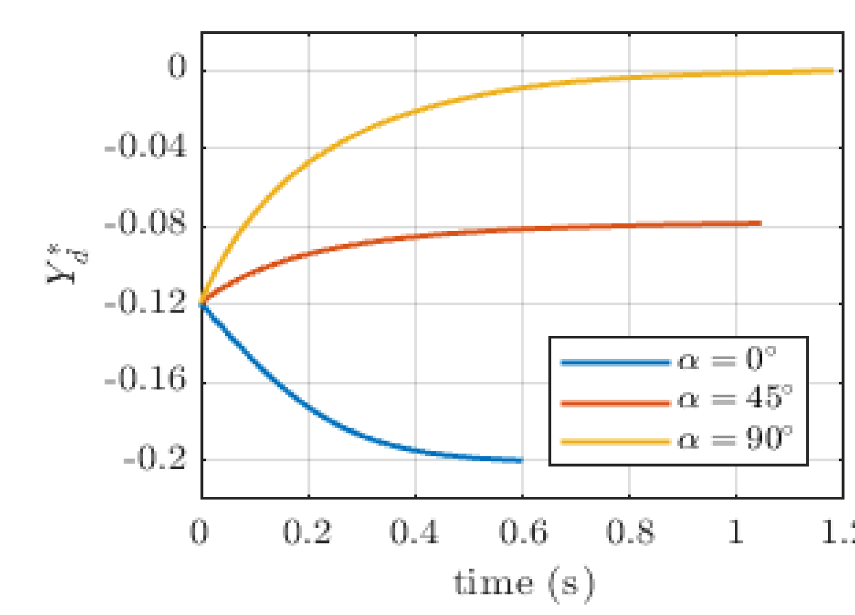

Figure 3. Effect of different magnetic field directions on the migration behavior of the droplet at $Re_d = 0.03, \lambda = 1,$ and $Bo_m = 8.72$.

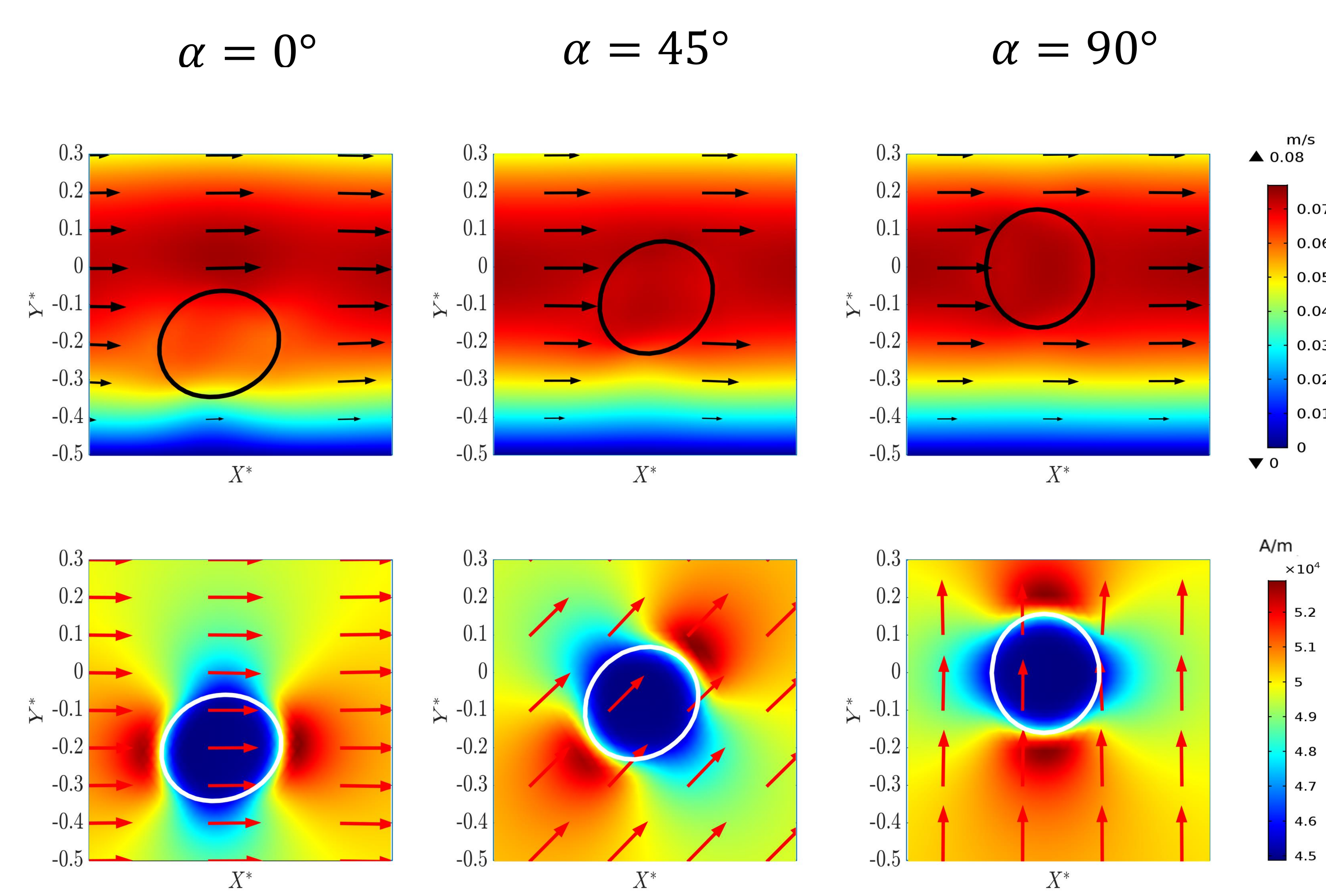

Figure 4. Steady state velocity, magnetic field profiles, and equilibrium droplet shapes at $Re_d = 0.03,\ \lambda = 1,$ and $Bo_m = 8.72$.

CONCLUSIONS: In the absence of any external forces, at $\lambda = 1$, the droplet finds its equilibrium position at a location approximately 19 µm below the center of the channel. Applying a magnetic field along arbitrary directions results in different equilibrium positions along the channel due to disparate alignments of the droplet with the flow field. At $\alpha = 0^{\circ}$, the droplet is found to be closer to the bottom wall, while at $\alpha = 45^{\circ}$ the droplet settles closer to the center, and at $\alpha = 90^{\circ}$, the droplet finds its equilibrium position exactly at the center of the channel.

References

1. COMSOL, "CFD Module Application Library Manual." .
2. E. Olsson and G. Kreiss, "A conservative level set method for two phase flow," *J. Comput. Phys.*, vol. 210, no. 1, pp. 225–246, 2005.