NUMERICAL AND EXPERIMENTAL STUDY ON ELECTROSTATIC PROPERTIES OF FRACTAL CAPACITORS

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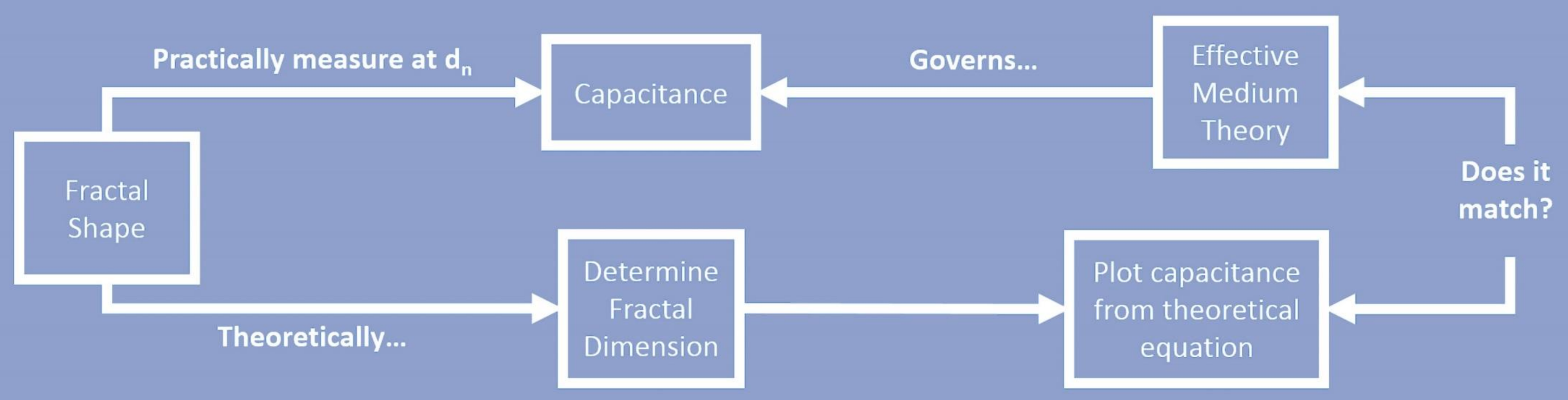
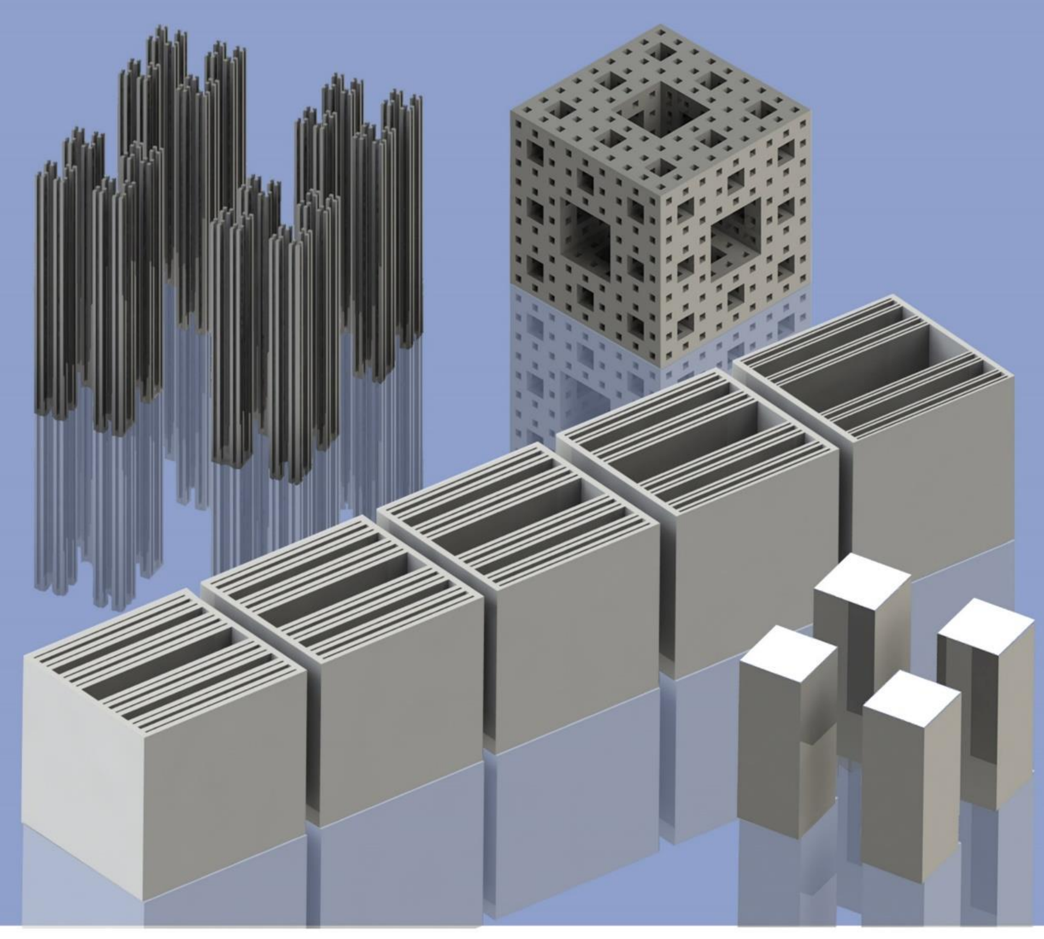




-> Solution to Laplace's equation for non-integer dimensions derived and applied to modelling heterogeneous media with fractal geometries.

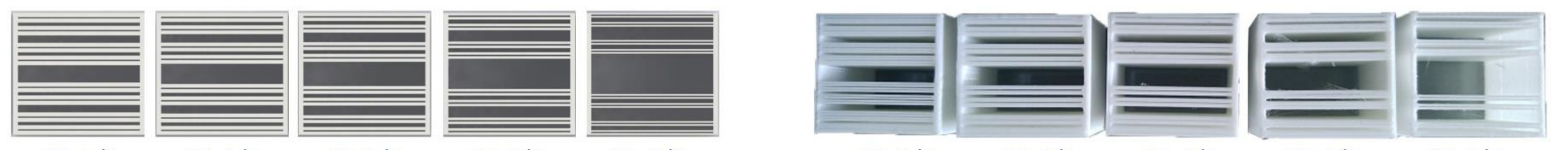
-> Fractal theory and effective medium theory can be applied to calculate fill factor of dielectrics, and hence permittivity and capacitance.

-> Conversely, by measuring capacitance at different distances or plate lengths, we can derive effective permittivity of the medium.

-> Finally, we check if the variation of C against L (or d) fits what our model predicts!

PRACTICAL EXPERIMENTS ON 3D-PRINTED CANTOR SETS

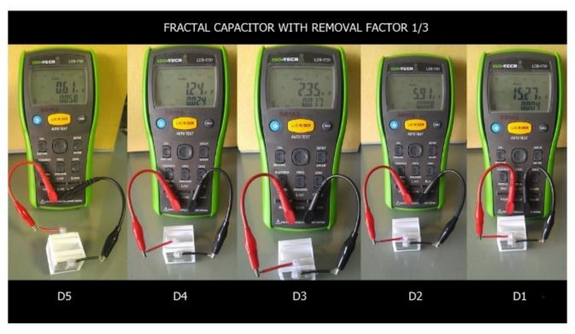
Cantor Sets 3D printed with PLA, relative permittivity of 3.8. Scatter plot of capacitance against distance is done to ascertain the value of alpha (dimension along fractal axis) experimentally. PLA dielectric cantor sets were printed as 30mm x 30mm x 30mm. Resolution of printing was performed at 200 microns. 5 different removal factors, one over: 3, 4, 5, 6, 7.



RF=1/3 RF=1/4 RF=1/5 RF=1/6 RF=1/7
3D Models of Cantor Sets on SolidWorks

RF=1/3 RF=1/4 RF=1/5 RF=1/6 RF=1/7
Actual 3D-prints of Cantor Sets using PLA filament

- Electrodes are aluminium plates, 25mm x 25mm.
- Measurements taken at positions of "d" where the fill factors of the dielectric are well known, and can be validated against both fractal theory and the effective medium theory.
- LCR meter used is ISOTECH LCR 1701 Handheld Meter with f=10kHz for greater precision at 0.01pF.



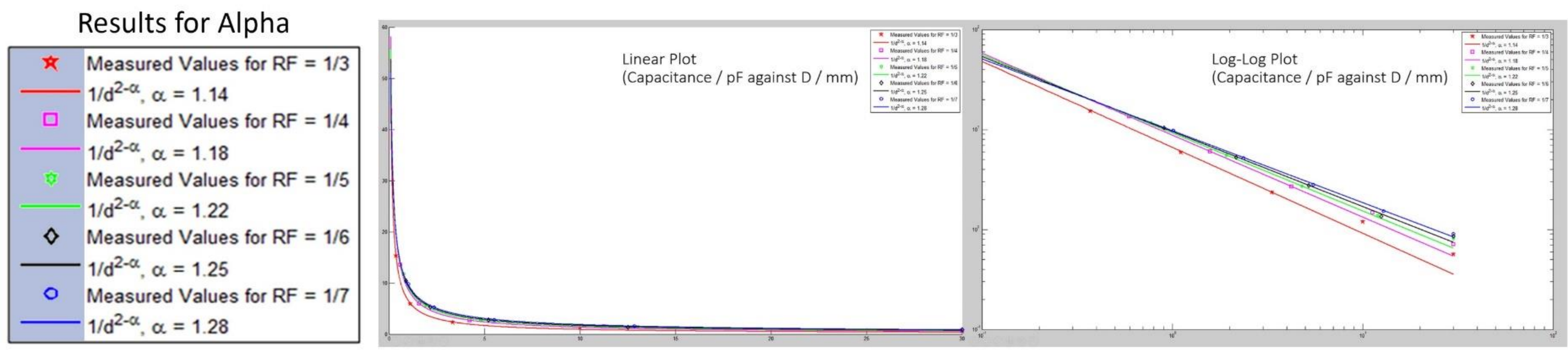
Results: Distance Measurement Points

	D1 / mm	D2 / mm	D3 / mm	D4 / mm	D5 / mm
RF: 1/3	0.37037	1.11111	3.33333	10.00000	30.00000
RF: 1/4	0.59326	1.58203	4.21875	11.25000	30.00000
RF: 1/5	0.76800	1.92000	4.80000	12.00000	30.00000
RF: 1/6	0.90422	2.17014	5.20833	12.50000	30.00000
RF: 1/7	1.01248	2.36244	5.51236	12.85714	30.00000

Results: Average Capacitance Values

	C1 / pF	C2 / pF	C3 / pF	C4 / pF	C5 / pF
RF: 1/3	15.2700	5.9100	2.3520	1.2000	0.5680
RF: 1/4	13.5780	6.0140	2.6900	1.4740	0.7100
RF: 1/5	11.6900	5.5340	2.7100	1.3820	0.8040
RF: 1/6	10.3920	5.2960	2.7680	1.3500	0.8580
RF: 1/7	9.7860	5.2000	2.8180	1.5340	0.9040

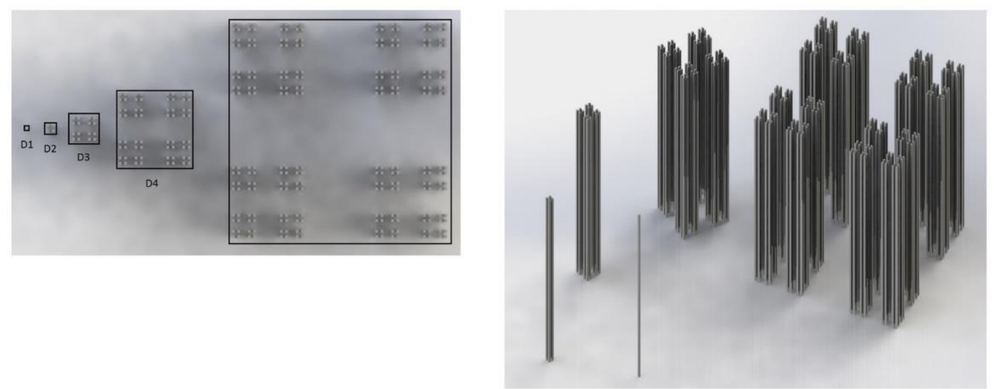
Results for Alpha



RESULTS: Alpha values increase with the denominator of removal factor as theory predicts. Howevr, all alpha values are > 1. Possible explanations: fringing effects due to finite area parallel plates, the imprecise thickness of dielectric layers due to resolution limits of 3D printing, the presence of stray capacitance from other sources, and also the higher sensitivity to changes in smaller pF values in log plot.

COMSOL SIMULATIONS ON CANTOR BARS

- Cantor Bars of RF = 1/3 are modelled.
- Dielectric constant of 3.8.
- Parallel electrode plates also fractal.
- Parallel plates across ends of dielectric.
- Capacitance recorded on COMSOL.
- Scatter plot of C/pF against length L/mm is validated against solution of Laplace's equation in non-integer dimensions.

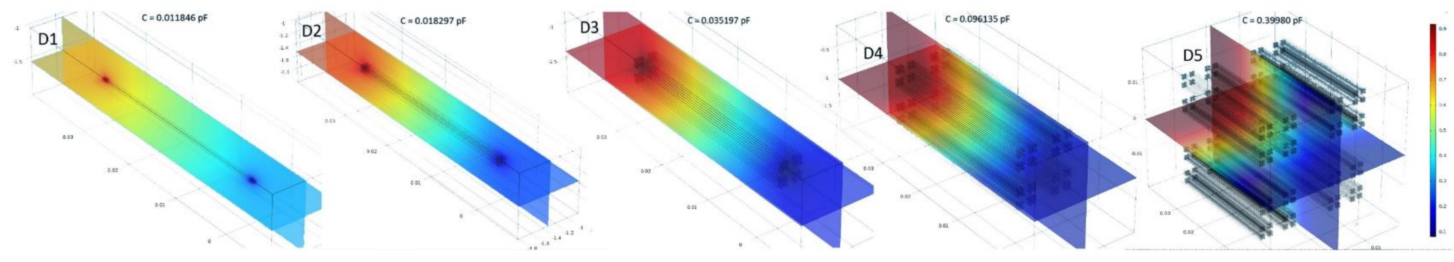


Equation Modelling

$$C = \varepsilon_o \left(\frac{3-\alpha}{2}\right)\left(\frac{4}{D^2}\right)\left(\frac{L^D}{d^{2-\alpha}}\right)$$

- ε_o is the permittivity of free space
- d is the plate separation distance between anode and cathode
- L is the plate length and width (they are equal since plate is square)
- D is the fractal dimension of the electrode plate (0 < D < 2)
- α is the dimension along plate separation distance 'd'

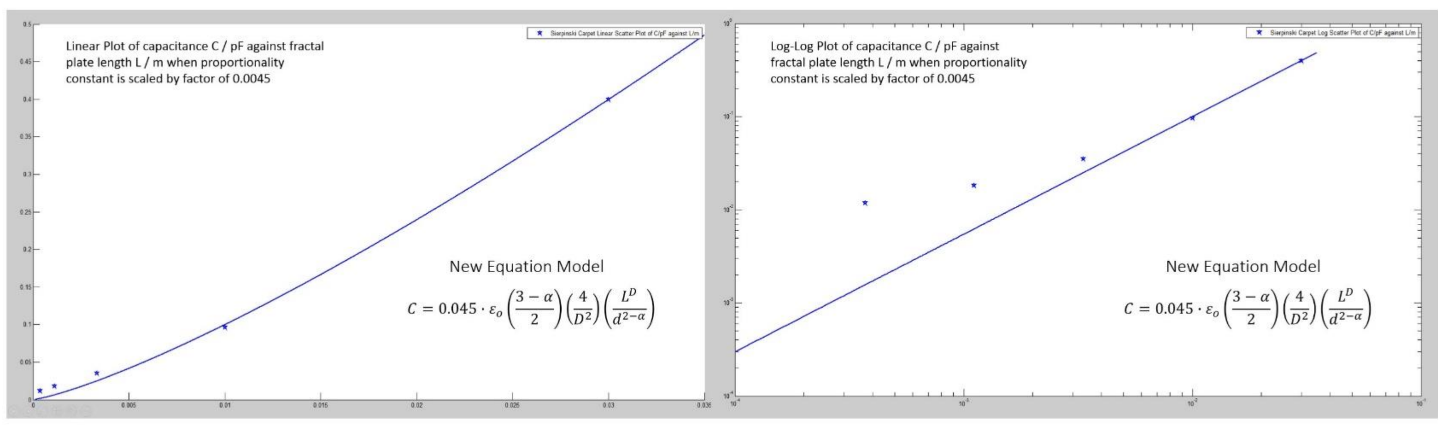
- In the case of Cantor Bars with Removal Factor of 1/3:
  - $\varepsilon_o = 8.85418782 \times 10E(-12)$
  - d = 0.030m
  - L is variable and possesses fractal dimension too
  - D = 1.2619 (Hausdorff Dimension)
  - α = 1, since no fractality along plate separation distance
  - $\therefore C = (7.413774 \times 10^{-10}) \cdot L^D$



- Voltage of 1V to GND was applied across the dielectric ends, and C/pF recorded against change in L/mm.

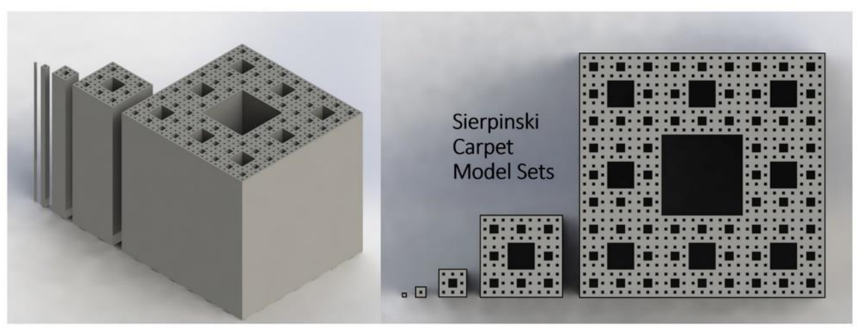
- Scaling was good but the fitting was not. For best fit, entire equation amplified by some coefficient of 0.045.

Cantor Bars of Removal Factor 1/3					
Iteration	1	2	3	4	5
L (mm)	0.37037	1.11111	3.33333	10.00000	30.00000
C (pF)	0.011846	0.018297	0.035197	0.096135	0.399800



COMSOL SIMULATIONS ON A SIERPINSKI CARPET

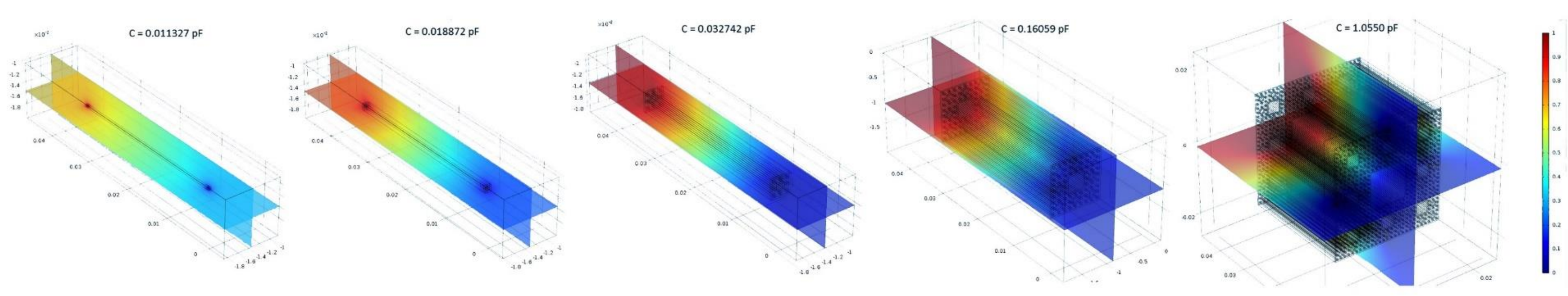
- Cantor Bars of RF = 1/3 are modelled.
- Dielectric constant of 3.8.
- Parallel electrode plates also fractal.
- Parallel plates across ends of dielectric.
- Capacitance recorded on COMSOL.
- Scatter plot of C/pF against length L/mm is validated against solution of Laplace's equation in non-integer dimensions.



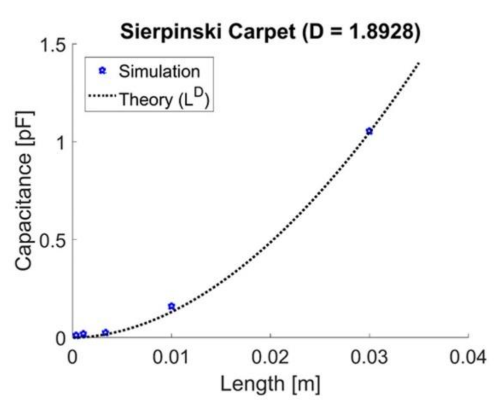
Equation Modelling

$$C = \varepsilon_o \left(\frac{3-\alpha}{2}\right)\left(\frac{4}{D^2}\right)\left(\frac{L^D}{d^{2-\alpha}}\right)$$

- In the case of Sierpinski Carpet:
  - $\varepsilon_o = 8.854 \times 10E(-12)$
  - d = 0.030m
  - L is variable and possesses fractal dimension too
  - D = 1.8926 (Theoretical Hausdorff Dimension)
  - α = 1, since no fractality along plate separation distance



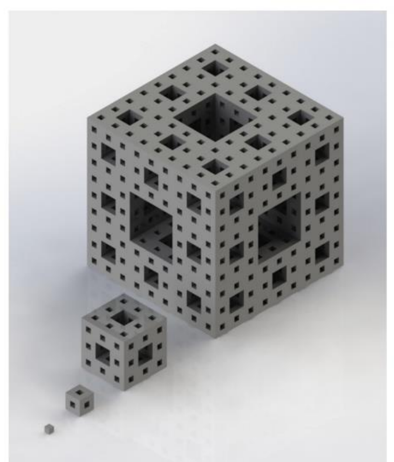
Results of Sierpinski Carpet					
Iteration	1	2	3	4	5
L (mm)	0.37037	1.11111	3.33333	10.00000	30.00000
C (pF)	0.011327	0.018872	0.032742	0.16059	1.0550



1V to GND was applied across the ends of the dielectric. Scaling was accurate (as shown in the logarithmic plots), especially with last three points. Curve was amplified by a factor of 2.55 for a better fit with the scatter plot.

COMSOL SIMULATIONS ON A MENGER SPONGE

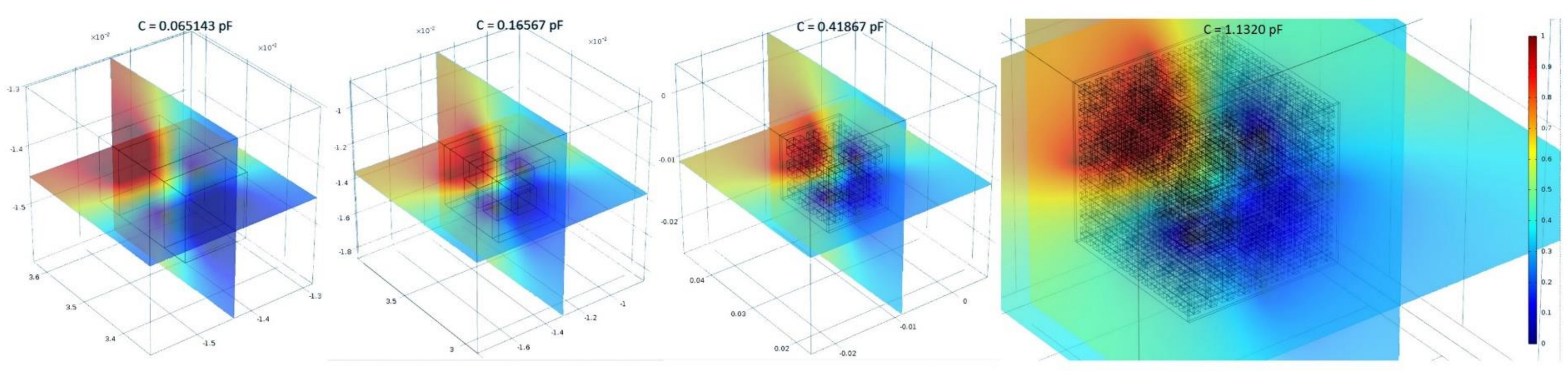
- Menger sponge, iteration n=3 is modelled.
- Dielectric constant of 3.8.
- Parallel electrode plates also fractal.
- Parallel plates across ends of dielectric.
- Capacitance recorded on COMSOL.
- Scatter plot of C against L validated against solution of Laplace's equation.



Equation Modelling

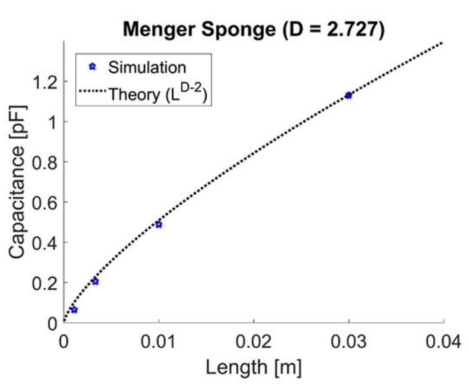
$$C = \varepsilon_o \left(\frac{3-\alpha}{2}\right)\left(\frac{4}{D^2}\right)\left(\frac{L^D}{d^{2-\alpha}}\right)$$

- In the case of Menger Sponge:
  - $\varepsilon_o = 8.854 \times 10E(-12)$
  - L = d = variable
  - Hausdorff Dimension = H_D =2.727
  - D = (2/3) * H_D
  - α = (1/3) * H_D
  - All 3 Cartesian axes exhibit similar fractality



Results of Menger Sponge				
Iteration	1	2	3	4
L (mm)	1.11111	3.33333	10.00000	30.00000
C (pF)	0.065143	0.16567	0.41867	1.1320

1V to GND was applied across the dielectric. Scaling was accurate, but not best-fit. Curve had to be amplified by factor of 1.2 for best fit line with the scatter plot.



FUTURE APPLICATIONS

Model can be used for modeling of anisotropic, inhomogeneous, disordered and fractal media where such applications exploit the usage of dielectric property detection in bio-imaging, scanning, microwave tomography and other electromagnetic applications, as many biological substances such as lung tissue exhibit fractal nature.

Model may also be useful to design dielectric heterostructures for engineering applications, e.g., fractal antennas, super-capacitors for energy storage applications, or porous low permittivity materials for next generation microwave integrated circuits.

REFERENCES

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