Numerical Simulation of Granular Solids' Rheology: Comparison with Experimental Results

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Abstract: The simulation of the behaviour of a bulk solid continuously flowing through a silo with internal flow feeders has been performed by means of a dissipative hydrodynamic model which has been already tested on simple geometries (flat bottomed silo, silo with converging hopper) showing good results [1].

The model will be now tested on more complex geometries, like silos with internal inserts useful to modify the flow profiles in some particular industrial applications with a continuous material flow through the silo.

This test will be carried out with a comparison between simulation and experimental results coming from an experimental campaign developed at Centro Ricerche Danieli.

The results obtained by these calculations and those found experimentally agree, not only with regard to the velocity profiles, but also relatively to the pressure profiles on silo's walls.

Keywords: Granular solids, CFD simulations, Rheology, Dense granular flow.

1. Introduction

The numerical models describing the flow of granular material like a fluid (thus resolving continuity and Navier-Stokes equations) assume for this pseudo-fluid non-Newtonian viscosity characteristics. Some of those, such as Bingham's or Cross' one relate viscosity to the shear rate [2,3].

The model used is similar, however it relates the viscosity to the shear rate of the bulk taking into account the particle mobility.

This is done through a scalar – called "granular temperature" (ϑ) for the similarity of its definition to thermodynamic temperature's one – which affects and is affected by the fluid's shear rate.

Granular temperature definition depends on particles velocity fluctuations and is therefore a measure of the local mobility of the medium.

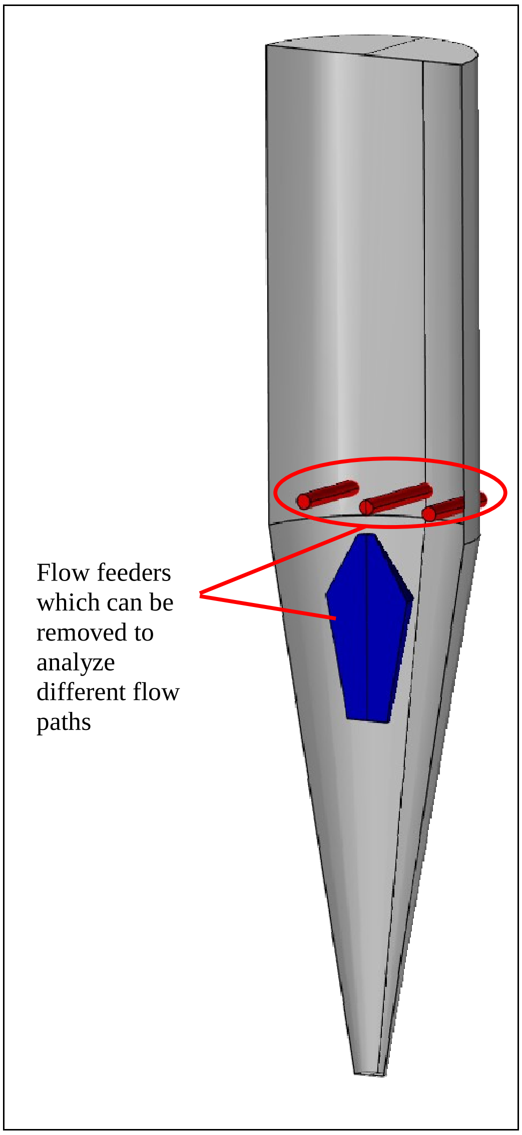

Figure 1. Schematic drawing of the experimental facility. Height ~3m, diameter ~0.7m, cone angle ~10°.

This model has been tested comparing the results obtained from the simulations with experimental ones in the case of a silo like the one in figure 1, existing at Centro Ricerche Danieli.

To study the flow patterns, the silo was only an half, cut vertically on a symmetry plane, and this plane was made of Plexiglas to observe the evolution of some tracer bands.

Some flow feeders were placed into the silo, to study different kinds of flow patterns and the reliability of the model. The flow feeders could be removed or placed in the silo as needed, generating four different kinds of geometries.

These internal devices were of two different types, as can be schematically seen in figure 1.

The first type was made of three tubes passing from side to side in the silo. The second one was more complex, made of two truncated cones one below the other.

2. Governing Equations

Writing the conservation equations and the constitutive laws for the variables considered, there is the need to make some assumption and approximations, which are:

- The interstitial fluid's effects are neglected
- The fluid is uncompressible
- No memory effects
- The stress tensor is symmetric.

Under these assumptions, the continuity equation becomes

$$\nabla \cdot \bar{\mathbf{v}} = 0$$

and the Navier-Stokes equation

$$\rho \frac{\partial \bar{\mathbf{v}}}{\partial t} + \rho \bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}} = -\nabla p - \nabla \cdot \Pi + \rho \mathbf{g}$$

where

$$\Pi_{ij} = -\eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$$

In addition the conservation equation for the granular temperature must be solved:

$$\rho \frac{\partial \vartheta}{\partial t} + \rho \bar{\mathbf{v}} \cdot \nabla \vartheta = -\Pi : \nabla \bar{\mathbf{v}} - \nabla \cdot \hat{\mathbf{q}}^{\mathbf{T}} - z^T$$

where $\hat{\mathbf{q}}^{\mathbf{T}}$ is a diffusive term for the scalar and z^T is the dissipative term due to the pressure that tends to block the particles limiting their movement.

The stress tensor Π can be related to the shear rate once a proper definition for the viscosity is given. This relationship is here of this type:

$$\eta = f(\vartheta).$$

Namely viscosity takes into account the particle mobility through an exponential function of ϑ.

3. Boundary Conditions

A peculiarity of granular materials is that they can slip at wall surface, contrary to classical fluids for which a zero velocity at wall boundaries is usually assumed.

The slip velocity is determined by the tangential stress the material is undergoing. For a granular material this stress can be described by the Coulomb condition

$$\tau = \mu_w \sigma$$

where μ_w is a dynamic wall friction coefficient and σ the normal stress. This condition showed some problem in the implementation into the software, so another kind of condition, called the Navier condition, has been used:

$$u_t = \lambda \left| \frac{\partial u_t}{\partial n} \right|$$

that relates the slip velocity u_t to its derivative along the wall normal vector n by means of a parameter (λ) called "slip length".

This has been done using for the wall boundaries the symmetry condition together with the weak formulation, inserting the shown condition in the boundary settings for the equation system.

Since experimentally the silo was open to air and discharged with a controlled mass flow rate, the boundary condition for the lower edge was a fixed velocity outlet and for the upper one was a pressure inlet with an atmospheric pressure. A constant material level is therefore assumed in the silo.

4. Experimental Method

The silo described in §1 and schematized in figure 1 was charged with polyethylene pellets with a 3mm mean diameter.

To study the material's flow profiles a tracer band made of differently coloured pellets was used and its position were recorded on camera at various times.

The silo was put under a steady state flow, with a continuous charging of material from the top to balance the extraction of solids from the hopper.

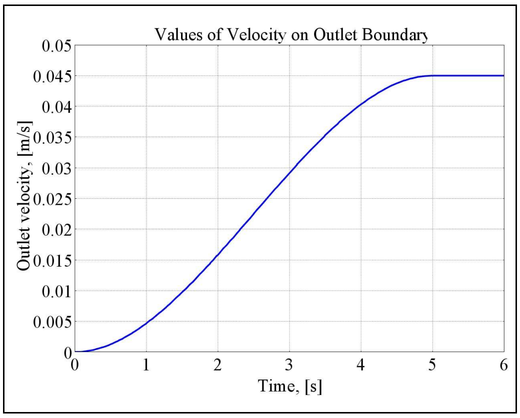

Figure 2. Example of the "soft start" function for the material velocity value set at the outlet boundary. Maximum velocity value 4.5 cm/s, t_0 5 seconds.

Wall stresses for the experimental facility were calculated according to theory (such as Jenike's [4]).

Various experiments were performed in different geometric configurations, with or without flow feeders one by one or together.

The first experiments, without flow feeders, were used to calibrate the model's parameters to obtain a good agreement with experimental results, while the others were useful for model's validation.

5. Numerical Simulations

The Multiphysics application modes used were the Incompressible Navier-Stokes (ns) from the base COMSOL Multiphysics module and the General Heat Transfer (htgh) from the Heat Transfer module.

Since the complexity of the model some precautions have been taken: the simulation were transient ones with a "soft start" function for the velocity, meaning that the discharge velocity value started from 0 and increased its value with time to reach the final one in a time t_0 which could vary case by case. An example of this function can be found in figure 2.

Another taken precaution was on the geometry; to reduce the number of degrees of freedom, an axysimmetric 2D model was used for these simulations. The simulated time span was the time of the entire experimental test, equal to 2 hours.

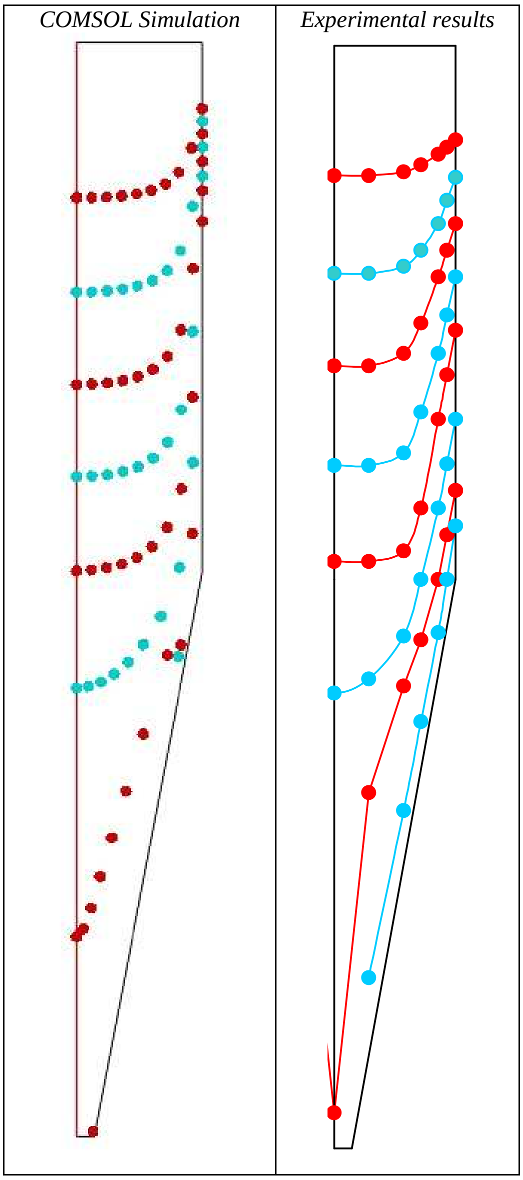

Figure 3. Comparison between tracer band profiles in simulation and in experiments. The time span between each profile is 15 minutes. The difference between the two profiles in the low zone it's due to the density changes not considered in simulation.

6. Results

As it was said before, the tests without flow feeders were first used to set up the model parameters while the others were used for validation.

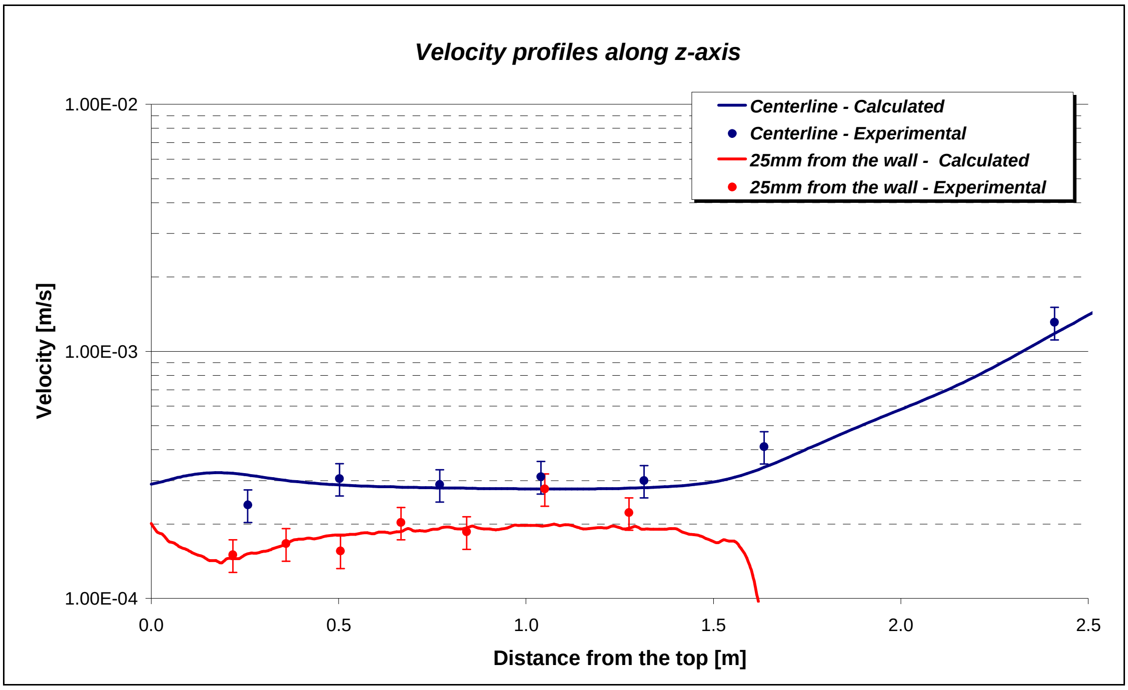

Figure 4. PE pellets velocity profiles along vertical direction for the base case. Error bars for experimental results ±15%.

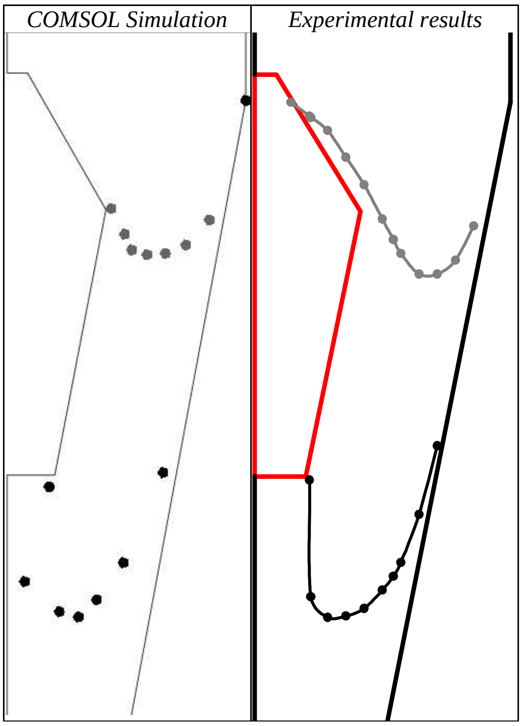

Figure 5. Tracer band descent profiles around the "double-cone" flow feeder.

These parameters were those entering in the definition of the fluid's viscosity and "granular temperature" and for the calculation of the slip velocity at walls.

6.1 Velocity and flow profiles

After the model calibration, the experimental and calculated flow profiles showed a good quantitative agreement, as can be seen in figure 3.

As it was said before, to study material's flow profiles a tracer band made of differently coloured pellets was used and its position were recorded on camera at various times.

Another kind of check that can be done to confirm the agreement between the results it's to analyze the fluid velocity along *z*-direction, as it's shown in figure 4.

After this calibration phase, calculations were performed for the different geometric configurations of the silo experimentally studied.

In figures 5 and 6 it is possible to see a comparison of the flow profiles around the flow feeders as calculated by COMSOL Multiphysics and as observed during experiments. It's noticeable that also in this cases simulations showed good quantitative agreement with experiments well reproducing the descent profiles in silo.

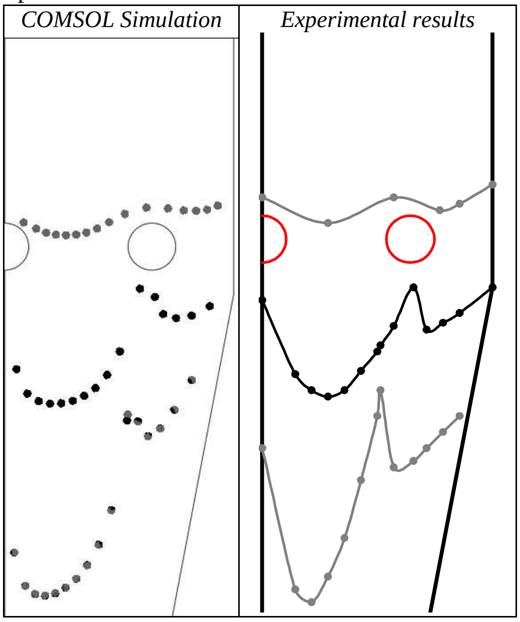

Figure 6. Solid's tracer band profiles around the cylindrical flow feeders.

6.2 Wall stresses profiles

Since no measurement of the wall stresses were performed during experiment, the comparison has been made between Multiphysics and calculations performed by theory.

The stresses in the hopper at discharge state are calculated due to a proposal of Arnold and McLean [5], where the theory of Jenike [4] is applied, while in the bin the stresses were computed starting from Janssen's slice element method [6].

Also in this case a good agreement between theory and simulation is shown (see Figure 6), especially in determining the stress peak at the transition between bin and hopper.

7. Conclusions

It has been shown that the dissipative hydrodynamic model [1] can well represent the behaviour of granular solids', not only with regard to the velocity and descent profiles of the material, but also as with respect to the stresses profiles at silo's walls.

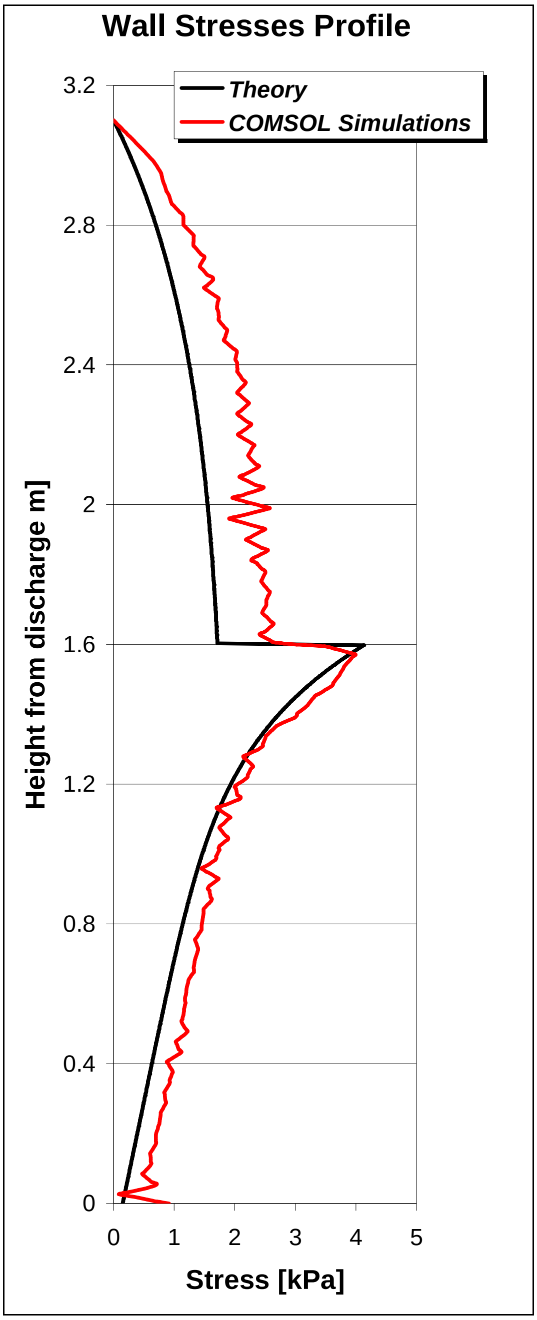

Figure 6. Walls normal stresses profiles in the silo without the flow feeders insertions.

This can be made not only on simple geometries [1] but also on more complex ones, like the ones simulated here with internal flow feeders.

Thus the model presented here can be very helpful both for the optimization of material flow profiles by means of internal geometry and for the design of the silo because of the good results in calculating wall stresses profiles.

Further improvements for the model could be the relaxation of some of the initial assumptions, such as not considering negligible the interstitial fluid and taking into account the density variations in the bulk during silo discharge phase.

This will be made after a new experimental campaign that will be developed at Centro Ricerche Danieli.

8. References

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