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The Non Linear Behaviour of the Microplane Model in COMSOL

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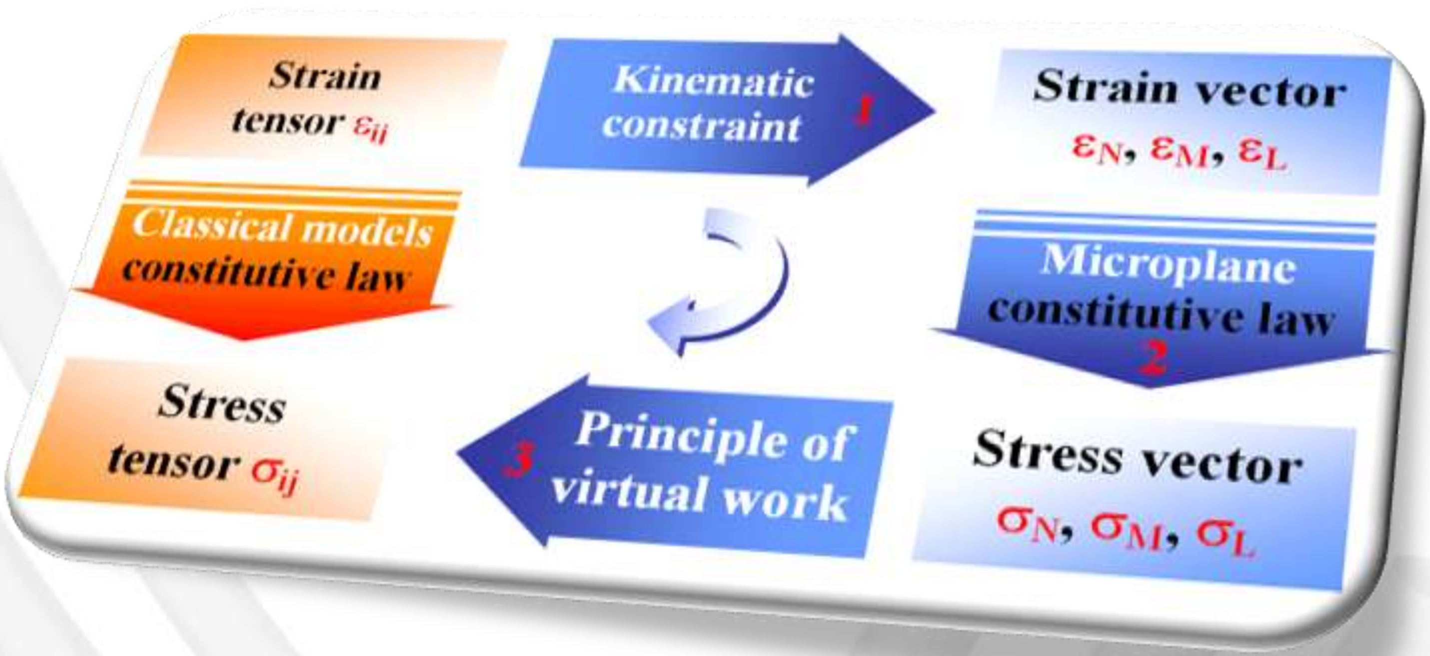
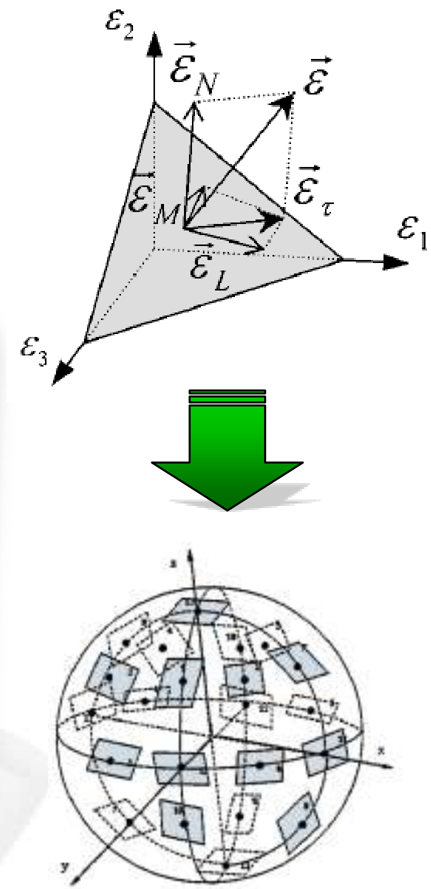
- Aims of the work
- The Microplane Model
  - A few hints on the main theory aspects
  - The non-linear behaviour
- Implementation process of the non-linear behaviour within COMSOL
- Conclusions

- Why the need to have another constitutive model for concrete?
  - Classical constitutive models are able to properly simulate *only a few specific characteristic* of concrete
  - The Microplane Model is a promising alternative approach able to simulate *the overall behaviour* of concrete





- *Logical scheme* of the linear elastic behaviour of the Microplane Model compared with that of classical approaches

- The *non-linear behaviour* is based on the definition of stress-strain boundaries at the microplane level:
  - Within the domain these boundaries mark out the material response is incremental elastic
  - Movements along these boundaries are permitted only if strain and stress increments have the same sign, otherwise elastic unloading occurs

- Damage can be modelled reducing progressively the elastic moduli of the incremental laws within the elastic domain
- The boundaries are characterized by 17 constant material parameters and 4 free parameters

$c_1, c_2 \ldots c_{17}$ & $k_1, k_2 \ldots k_4$

- The constant parameters should be kept fixed for all types of concrete
- The free parameters should be identified fitting test data

Tensile normal boundary

$$\sigma_N = \sigma_V + \sigma_D$$

Volumetric boundaries

$$\dot{\sigma}_V = E_V \dot{\varepsilon}_V$$

Deviatoric boundaries

$$\dot{\sigma}_D = E_D \dot{\varepsilon}_D$$

Frictional yield boundary

$$\sigma_T = \sqrt{\sigma_M^2 + \sigma_L^2}$$

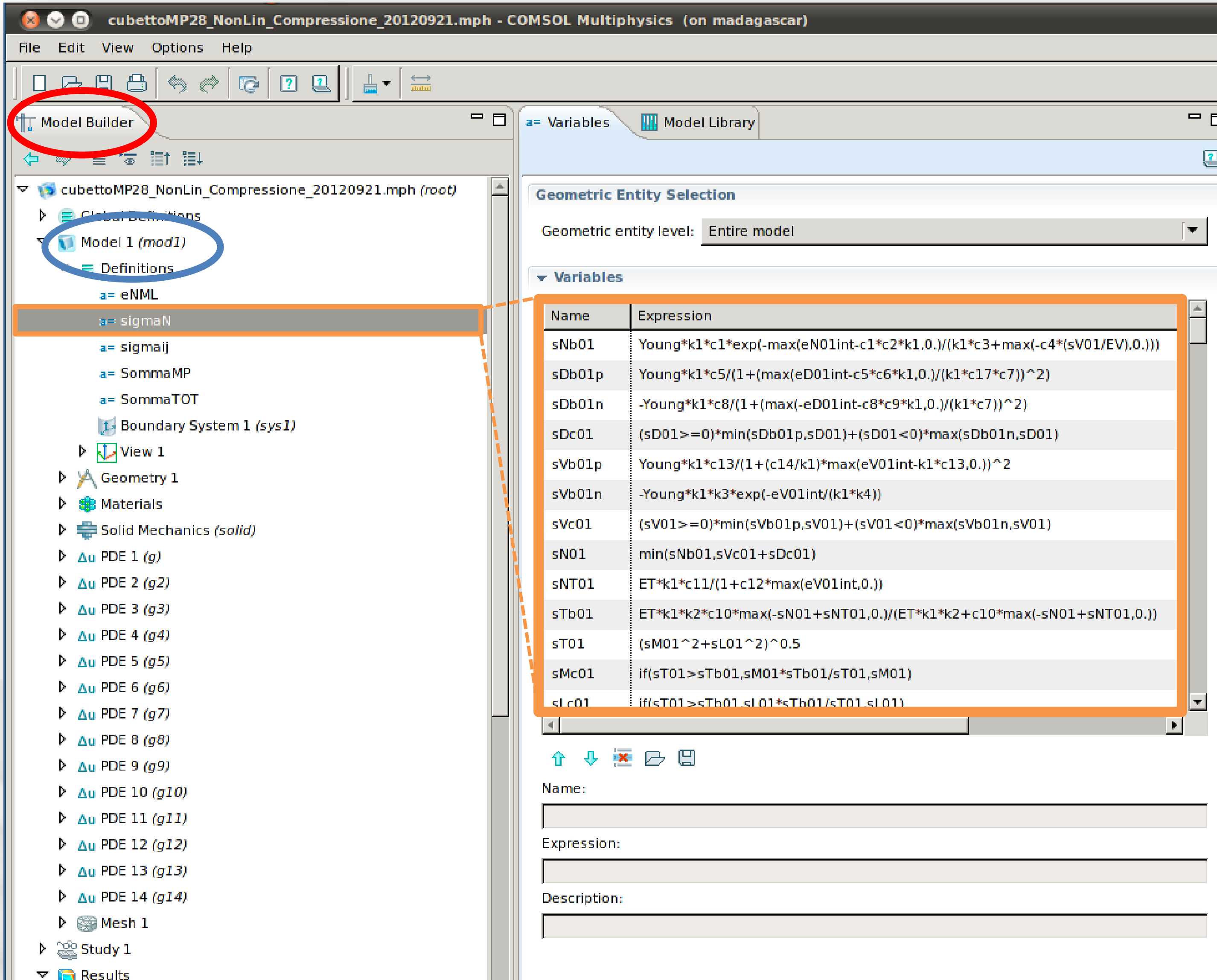
$$\dot{\sigma}_M = E_M \dot{\varepsilon}_M \qquad \dot{\sigma}_L = E_L \dot{\varepsilon}_L$$

Implementation within COMSOL

Model Builder window

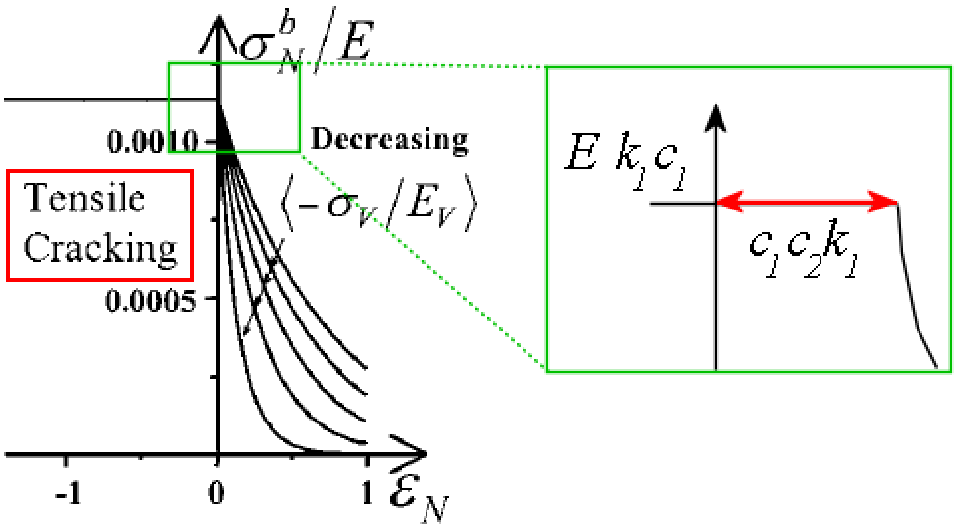
Model Definitions node

SigmaN



Tensile normal boundary

$$\sigma_N^b = E k_1 c_1 \exp\left(- \frac{\langle \varepsilon_N - c_1 c_2 k_1 \rangle}{k_1 c_3 + \langle - c_4 (\sigma_V / E_V) \rangle} \right)$$

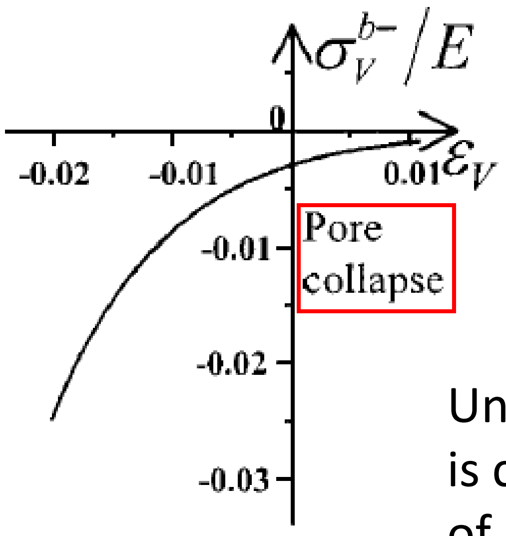


- The initial descending part describes the tensile cracking parallel to the microplane
- The tail defines the frictional pullout of fragments bridging the crack surfaces

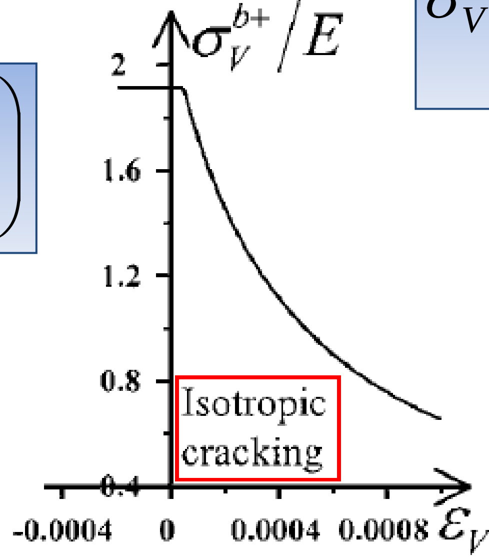
```
sNbkk = Young*k1*c1*exp(-max(eNkkint-c1*c2*k1,0.)/(k1*c3+max(-c4*(sVckk/EV),0.)))

sNkk = min(sNbkk,sVckk+sDckk)
```

Volumetric boundaries

$$\sigma_V^b = -Ek_1k_3\exp\left(-\frac{\varepsilon_V}{k_1k_4}\right)$$



$$\sigma_V^b = \frac{Ek_1c_{13}}{\left[1+(c_{14}/k_1)\langle\varepsilon_V - k_1c_{13}\rangle\right]^2}$$



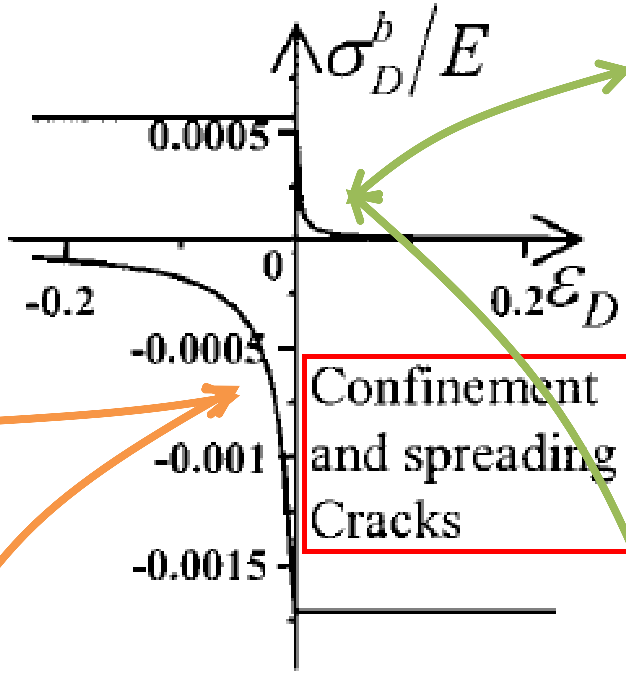
A tensile volumetric boundary is needed to prevent unreasonable lateral strains in post peak softening under uniaxial, unconfined, tension

Under hydrostatic pressure a progressive stronger hardening is considered to primarily represent the collapse and closure of pores

```
sVbkkn = -Young*k1*k3*exp(-eVkkint/(k1*k4))
sVbkkp =  Young*k1*c13/(1+(c14/k1)*max(eVkkint-k1*c13,0.))^2

sVckk  = (sVkk>=0)*min(sVbkkp,sVkk)+(sVkk<0)*max(sVbkkn,sVkk)
```

Deviatoric boundaries

The compressive deviatoric curve controls the axial crushing strain of concrete in compression when lateral confinement is too weak to prevent crushing



The tensile deviatoric curve:

- simulates transverse crack opening of axial distributed cracks in compression
- controls the volumetric expansion and lateral strains in unconfined compression tests

$$\sigma_D^b = -\frac{E k_1 c_8}{1 + \left(\langle -\varepsilon_D - c_8 c_9 k_1 \rangle / k_1 c_7 \right)^2}$$

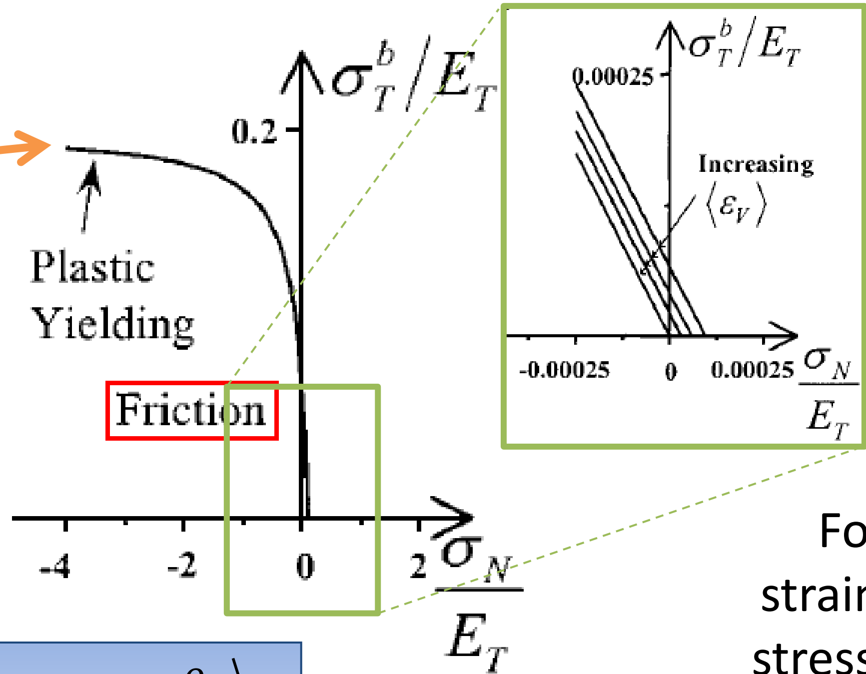
$$\sigma_D^b = \frac{E k_1 c_5}{1 + \left(\langle \varepsilon_D - c_5 c_6 k_1 \rangle / k_1 c_{17} c_7 \right)^2}$$

```
sDbkkn = -Young*k1*c8/(1+(max(-eDkkint-c8*c9*k1,0.)/(k1*c7))^2)
sDbkkp =  Young*k1*c5/(1+(max(eDkkint-c5*c6*k1,0.)/(k1*c17*c7))^2)

sDckk =  (sDkk>=0)*min(sDbkkp,sDkk)+(sDkk<0)**max(sDbkkn,sDkk)
```

Frictional yield boundary

$$\lim_{\sigma_N \to -\infty} \sigma_T = E_T k_1 k_2$$

At very high confining pressures, concrete becomes a plastic but frictionless material



$$\left[\frac{d\sigma_T}{d\sigma_N}\right]_{\sigma_N=0} = -c_{10}$$

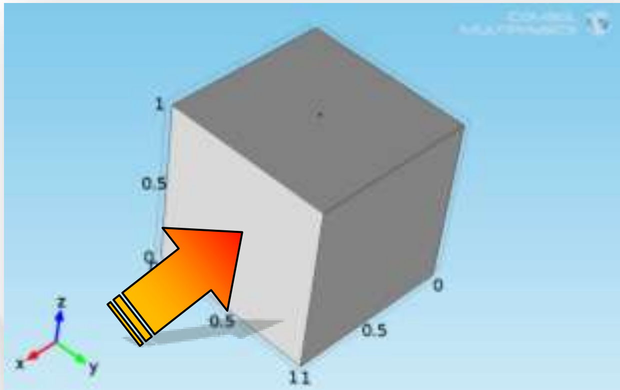
$$\sigma_T^b = \frac{E_T k_1 k_2 c_{10} \left\langle -\sigma_N + \sigma_N^0 \right\rangle}{E_T k_1 k_2 + c_{10} \left\langle -\sigma_N + \sigma_N^0 \right\rangle}$$

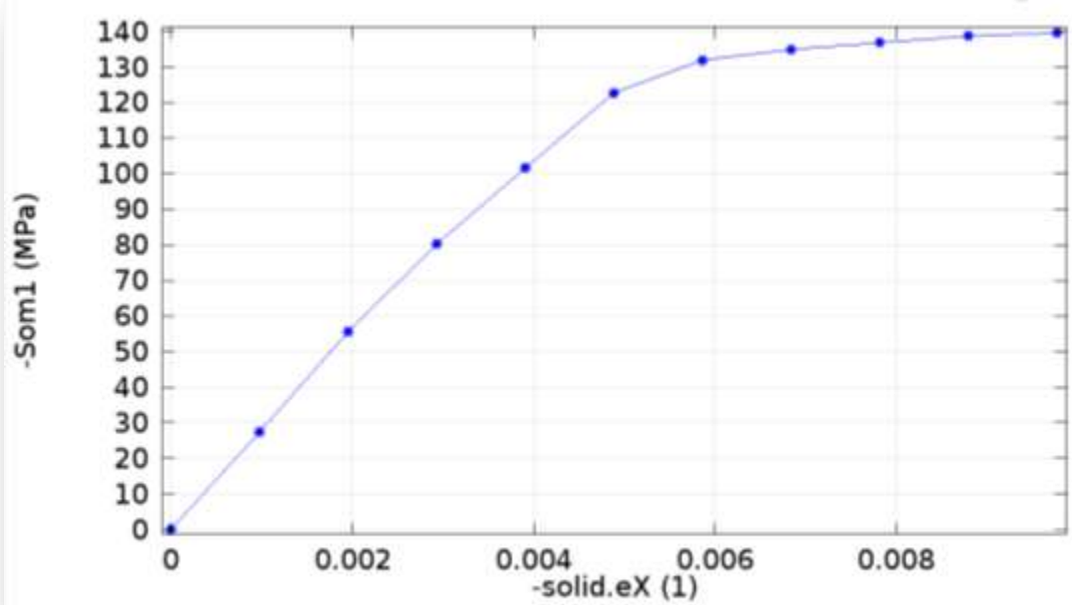
For small volumetric strain, a finite cohesive stress, which decreases to zero with increasing volumetric strain, is provided

$$\sigma_N^0 = \frac{E_T k_1 c_{11}}{1 + c_{12} \langle \varepsilon_V \rangle}$$

```
sNTkk = ET*k1*c11/(1+c12*max(eVkkint,0.))
sTbkk = ET*k1*k2*c10*max(-sNkk+sNTkk,0.)/(ET*k1*k2+c10*max(-sNkk+sNTkk,0.))

sTkk  =(sMkk^2+sLkk^2)^0.5
sMckk = if(sTkk>sTbkk,sMkk*sTbkk/sTkk,sMkk)
sLckk = if(sTkk>sTbkk,sLkk*sTbkk/sTkk,sLkk)
```

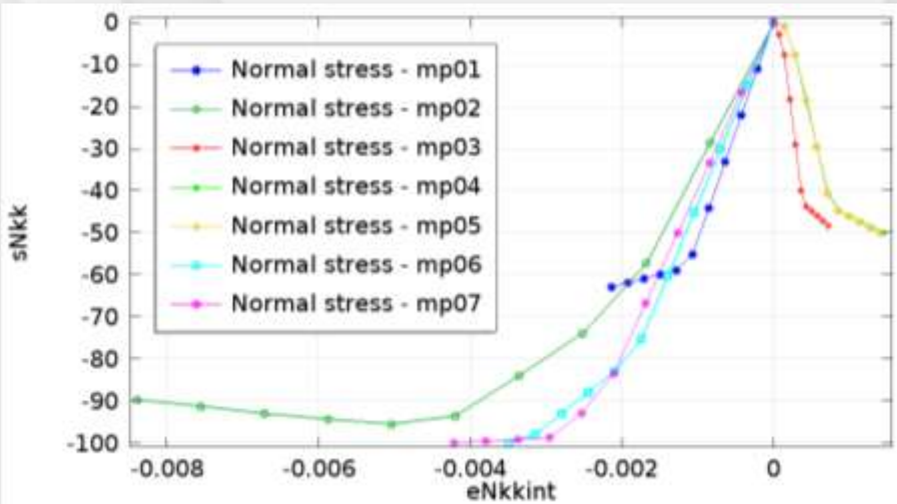
Validation of the model

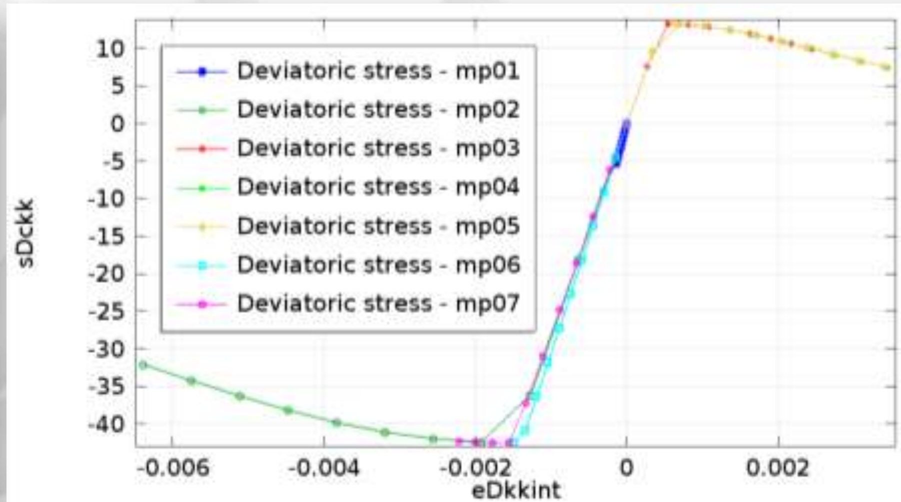




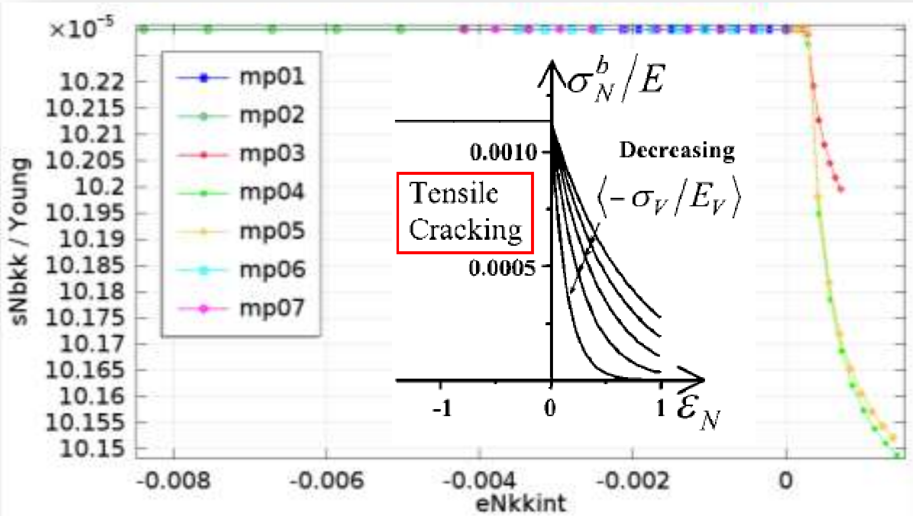
Compression test

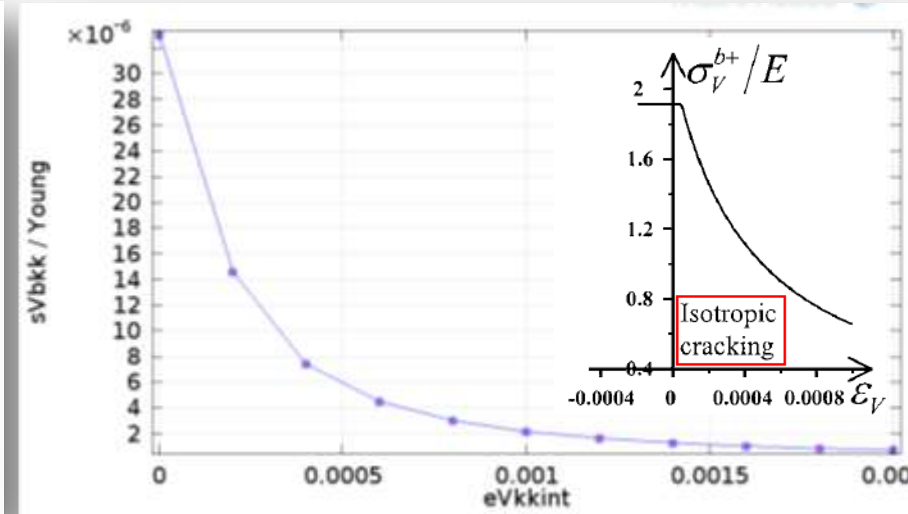
Results in terms of stress

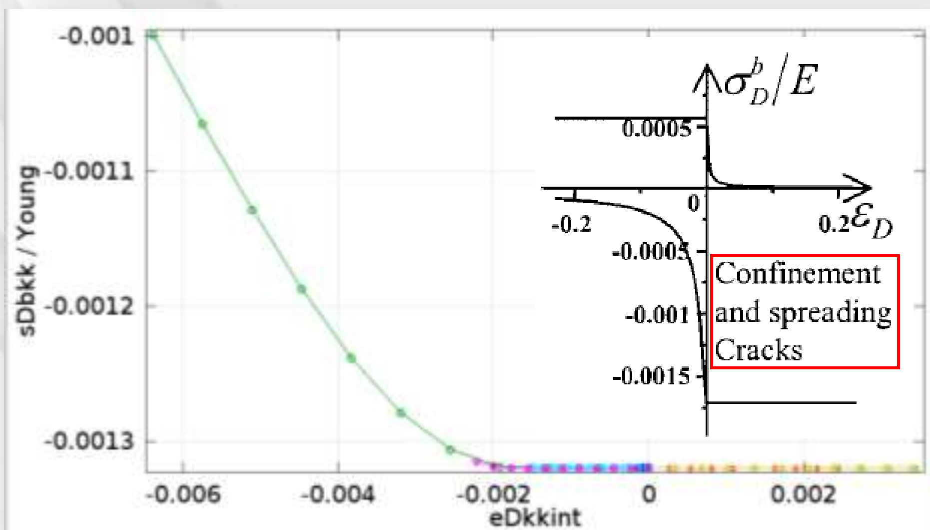


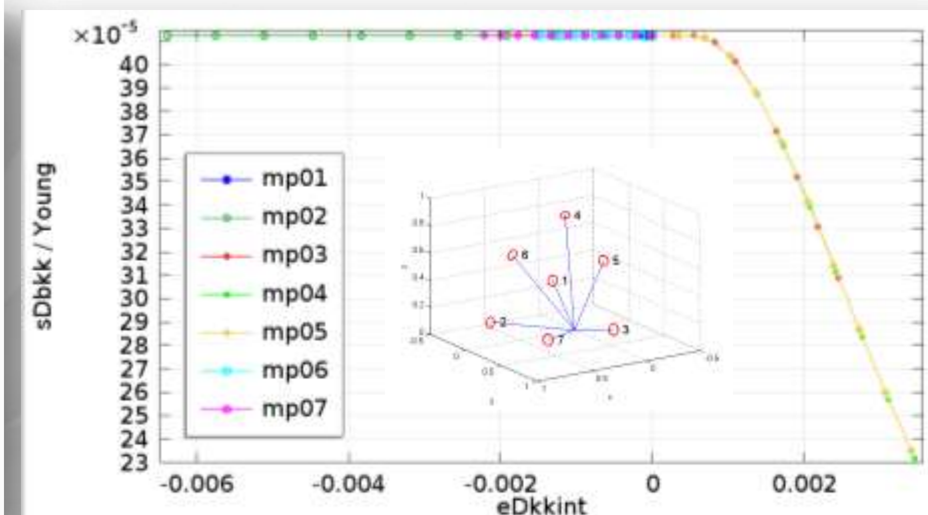


Non-linear boundaries check during a compression/traction test









- An accurate identification of the free material parameters and the constant ones is needed
- Applying the Microplane Model to simulate the concrete behaviour of large structures, such as dams, presenting an evident crack pattern

Thank you for your attention

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