Numerical Study of the Scattering of a Short-Pulse Plane Wave by a Buried Sphere in a Lossy Medium

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Abstract: The scattering by a buried sphere in the frequency domain with the use of the Finite Element Method (FEM) implemented by COMSOL Multiphysics, is analyzed. A short-pulse is used as an excitation with the spectrum spanning from 50 MHz to 1 GHz. In order to validate our results, a comparison with data available in the literature is presented, in the simple case of a perfectly-conducting (PEC) sphere. Afterwards, to gain insight on the role of the sphere radius and the distance of the buried sphere from the interface, other simulations are performed. The case of a dielectric sphere, instead of the perfectly-conducting one, is studied too.

Keywords: electromagnetic scattering, buried objects, lossy media, FEM.

1. Introduction

Exploiting the scattered electromagnetic waves from underneath the soil, the Ground Penetrating Radar (GPR) is a device that can detect the presence of objects buried in the earth [1]. For the study of GPR systems a great variety of numerical methods, from analytical approaches to time and frequency domain algorithms, have been employed [2].

Since other approaches, or variations of the above methods, are being adopted by our group to study the electromagnetic field scattered by buried objects, additional and reliable tools are essential to validate our methods.

Believing that COMSOL’s RF module may be used to study the field solution for the impinging radiation on buried objects, the canonical problem of the scattering of a plane wave by a buried sphere in a lossy medium, which has been extensively studied in the past, has been modeled and validated: the comparison of the solution obtained by COMSOL with the data available in literature [3] offers a means to assess the accuracy of the simulations results.

In Section 2 the statement of the problem and the setting of the COMSOL model are presented. The simulation results and their validation are shown in Section 3. Finally, in Section 4 conclusions are drawn.

2. Statement of the problem

A sphere with radius a is buried in a half-space filled by a dispersive lossy medium at a distance h from the interface. We choose the origin of the referring frame to coincide with the center of the sphere as shown in Fig. 1.

The excitation considered is a short-pulse with a spectrum spanning from 50 MHz to 1 GHz (Fig. 2), formed by a linearly s-polarized plane wave impinging normally to the interface, i.e., $\vartheta_i=\varphi_i=0$. The dispersion characteristics of the soil, supposed to be slightly wet clay, are shown in Fig. 3. This excitation is taken into account to be consistent with [3].

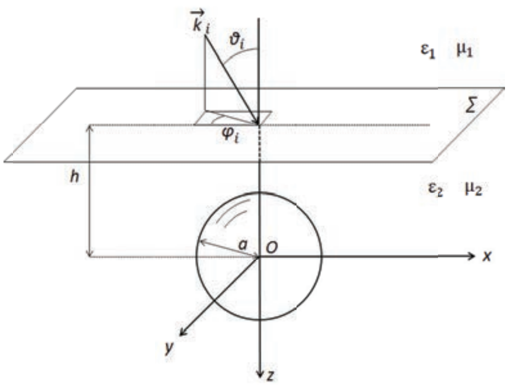

Figure 1. Geometry of the problem.

The RF module's scattered field formulation is used. The background field is analytically specified as follows:

$$\underline{E}_b(\underline{r}) = \begin{cases} \underline{E}_0 e^{i\underline{k}_i\cdot\underline{r}} + R\underline{E}_0 e^{i\underline{k}_r\cdot\underline{r}} e^{i\varphi_r} & z < -h \\ T\underline{E}_0 e^{i\underline{k}_t\cdot\underline{r}} e^{i\varphi_t} & z > -h \end{cases}$$

where R and T are the Fresnel reflection and transmission coefficients, respectively.

Furthermore, φ_r and φ_t are the reflection and transmission phase shifts, respectively, due to the fact that the position of the origin of the reference frame is not on the interface. On the other hand, since the ground material is dispersive, the characteristics of the medium are implemented in COMSOL interpolating the experimental data [4].

A spherical domain is chosen with a spherical PML centered on the projection of the reference frame origin on the interface. This is due to the fact that the scattered reflected/transmitted waves behave as spherical waves originating from this projection point. The maximum dimension of the mesh cells is $\lambda/5$ and we use a swept mesh for the PML region. It is important to note that λ refers to the wavelength in the considered medium, i.e., the vacuum wavelength divided by the refractive index.

The problem is solved by performing a frequency sweep.

3. Results

In the following subsections, we present the validation of our simulation model and the study of the dependence of the scattered field on the sphere radius and its depth.

3.1 Comparison with data by Vitebskiy et al.

Vitebskiy et al., in [3], apply the method of moments (MoM) to the study of a perfectly-conducting sphere of radius 15 cm, buried in dry clay at a depth of 30 cm. The magnitude of the co-polarized scattered electric field is observed at the air-clay interface directly above the center of the PEC sphere. We take these data as a reference.

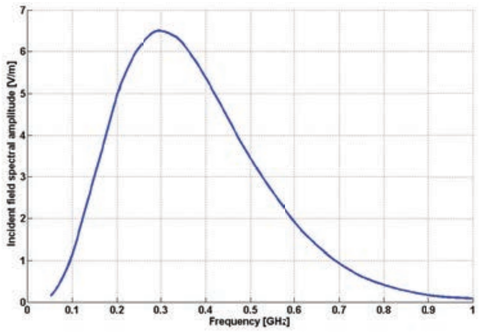

Figure 2. Spectrum of the incident short-pulse field.

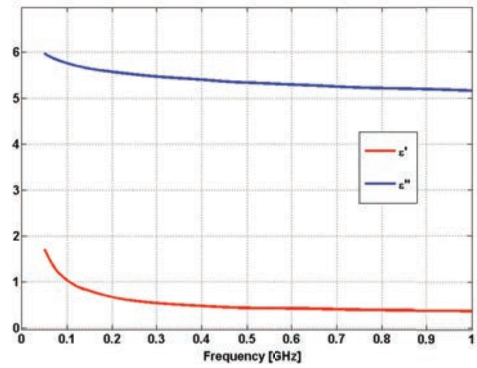

Figure 3. Relative complex permittivity $\varepsilon=\varepsilon'+i\varepsilon''$ measured for clay with 10% water content.

A comparison of the results obtained by COMSOL simulations and the aforementioned study is shown in Fig. 4. As can be observed, an excellent agreement between the COMSOL results and the reference data exists in the entire frequency range of interest.

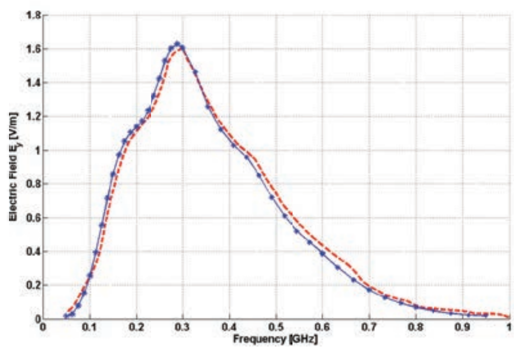

Figure 4. Comparison of the results obtained by COMSOL (continuous blue line) and those of Vitebskiy et al. [3] (dashed red line), for $a=15$ cm and $h=30$ cm.

We note that for this specific problem we had to split the frequency sweep in three different intervals to overcome mesh optimization problems.

3.2 Simulation results

Having established the validity of our model, we proceed to consider some special cases in order to point out the effects of the variation of the geometrical parameters on the scattered field. The results presented in Figs. 5-8 are related to the field value at the projection point of the reference frame origin on the interface. All of the results refer to an incident plane wave linearly-polarized along *y*, with center frequency $f_0 = 300$ MHz and amplitude 1 V/m, impinging normally on an air-glass interface ($\varepsilon_{air} = 1$, $\varepsilon_{glass} = 2.25$). The radiation wavelength is therefore roughly 1 m in air and 0,67 m underneath the soil surface.

Initially we study the dependence of the scattered field with respect to the radius and depth of a perfectly conducting sphere. In Fig. 5, the absolute value of the scattered field versus the sphere radius is plotted for a depth of 30 cm. As the sphere radius grows, the scattered field monotonically grows because, at a fixed depth, the field probe is near the scatterer.

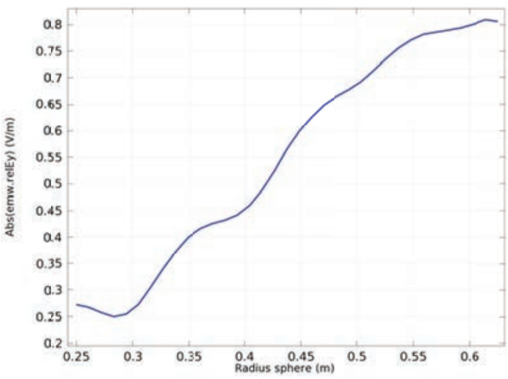

Figure 5 Absolute value of the relative scattered electric field along *y* by a PEC sphere. Results obtained for $h=30$ cm and a radius varying from $a_{min} = 25$ cm to $a_{max} = 2.5a_{min}$.

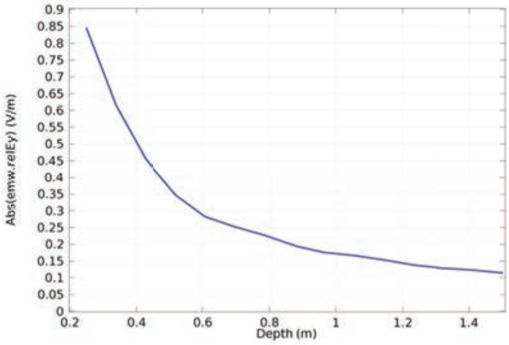

Figure 6 Absolute value of the relative scattered electric field along *y* by PEC sphere results obtained for the radius $a=25$ cm and the depth from $h_{min}=a$ to $h_{max}=6a$.

The slight deviation from the ideal monotonic behavior is probably due to the presence of the interface. Furthermore in Fig. 6, the results of a fixed radius sphere (a=25 cm) at varying depth are shown. It can be noted that the absolute value of the scattered field, as expected, is inversely proportional to the depth *h*. As the electromagnetic field penetrates deeper into the soil, the scattered portions lower. This effect may be explained considering the sphere surface being constant with respect to depth variations, taking into account that the scattered field is close to a spherical one.

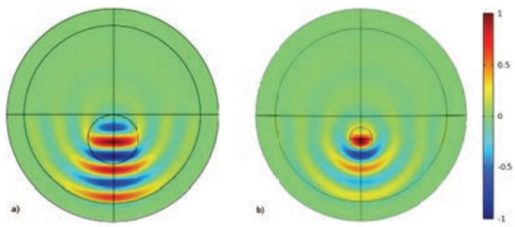

Figure 7 Normalized scattered electric field by an air sphere on the plane $y=0$ for a) $a = a_{max} = 62.5$ cm; b) $a = a_{min} = 25$ cm.

The other case we consider is an air sphere embedded in glass. For illustration purposes, in Fig. 7 we plot the normalized scattered field in the two cases of minimum and maximum radius, $a_{max}=62.5$ cm and $a_{min}=25$ cm respectively.

Similarly to the previous case, in Fig. 8 we present the dependence of the scattered field on the sphere radius.

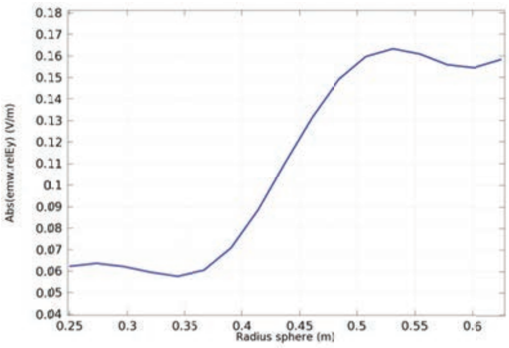

Figure 8 Absolute value of the relative scattered electric field along *y* by an air sphere ($\varepsilon_{air}=1$), results obtained for $h=30$ cm and the radius from $a_{min}=5$ cm to $a_{max}=62.5cm$.

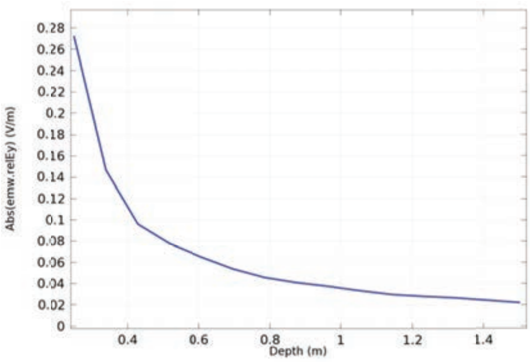

Figure 9 Absolute value of the relative scattered electric field along *y* by air sphere ($\varepsilon_{air}=1$), results obtained for the radius $a=25$ cm and the depth from $h_{min}=a$ to $h_{max}=6a$.

The behavior of the scattered field with respect to the sphere depth at a fixed radius is similar to the previous case.

4. Conclusions

The simulation results provided by the COMSOL model used to calculate the field scattered by a buried PEC sphere are in very good agreement with previously published data. The RF module has been found to be a versatile and efficient tool for the study of the canonical problem of a buried sphere in a lossy medium in different configuration schemes. In future works, we plan to use the Transient Electromagnetic Waves Interface for the study of these canonical problems to compare the results obtained by employing other methods, both analytical and numerical.

5. References

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