COMSOL simulations to develop and investigate the efficiency of a rocket-borne particle collector

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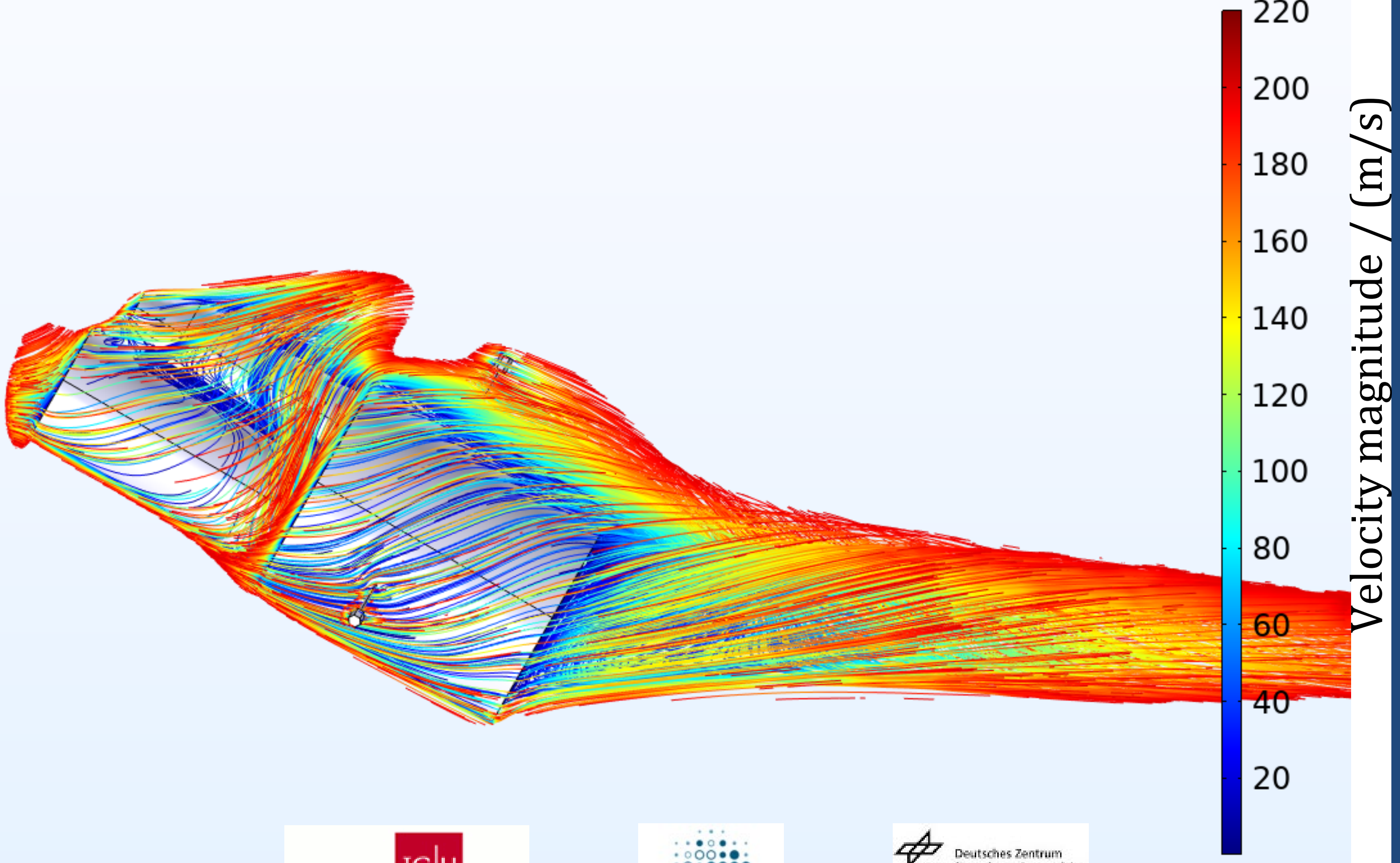

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MAX-PLANCK-INSTITUT FÜR CHEMIE

DLR Deutsches Zentrum für Luft- und Raumfahrt German Aerospace Center

MORABA MOBILE RAKETENBASIS

COMSOL MULTIPHYSICS®

Altitude:
~85 km

Idea:

Sampling based, free-stream impactor

- **Collection of particles on the surface of a flow obstacle**
- **Analysis of the particles**

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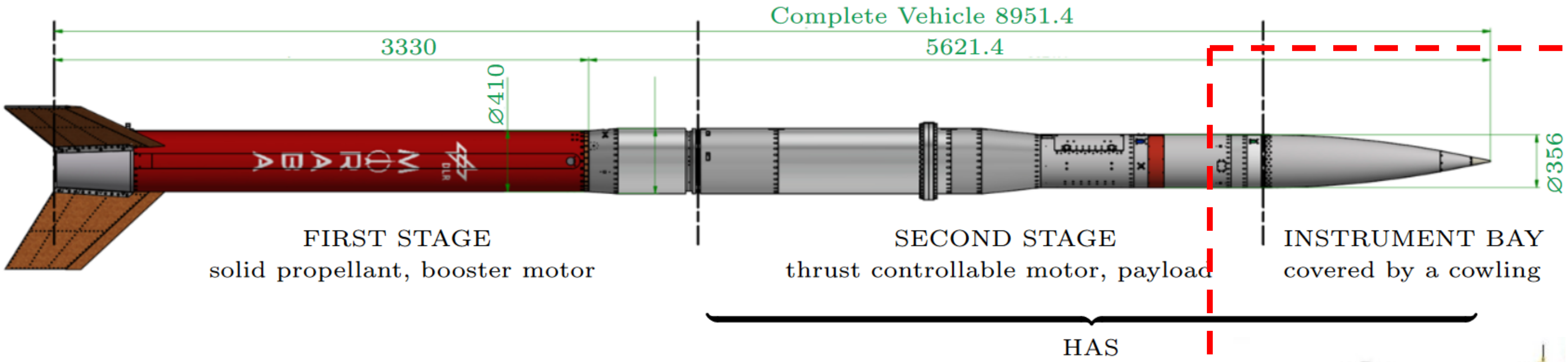
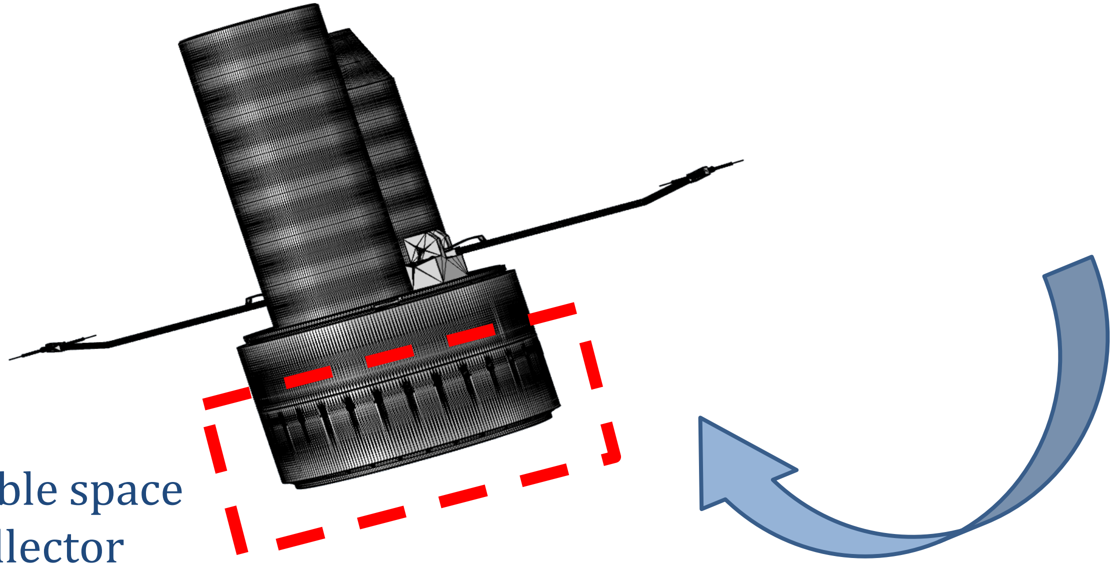
05 Jul 2020, Sulzheim

Rocket



instrument module



the available space for the collector

adopted from Naumann, K., et al. "Design of a hovering sounding rocket stage for measurements in the high atmosphere." (2020)

Mathematical and numerical model of supersonic flows around the instument module

Supersonic flow regime

rocket speed

$v = 300 - 400\,\frac{\mathrm{m}}{\mathrm{s}}$

speed of sound

$c_{85\ \mathrm{km}} \approx 230\,\frac{\mathrm{m}}{\mathrm{s}}$

Mach number

$Ma = 1.3 - 1.8$

High Mach Number Flow, Laminar (hmnf) interface ( )

Compressible Navier Stokes equations

continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \vec{u}) = 0$$

momentum equation

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}^T) = \nabla \cdot \mathrm{T_f} + \rho \vec{f}$$

energy equation

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\rho \vec{u} E) = \, - \nabla \cdot k \nabla T + Q + \nabla \cdot (\mathrm{T_f} \vec{u}) + \rho(\vec{f} \cdot \vec{u})$$

internal energy equation
ideal gas law

$$\text{with } E = e + \frac{u^2}{2}, \text{ and } e = c_v\, T, \rho = \frac{p}{\mathrm{R_s}\, T},$$

$$\mathrm{T_f} = -p\mathbf{I} + \left[\mu\left(\nabla \vec{u} + (\nabla \vec{u})^T - \frac{2}{3}(\nabla \cdot \vec{u})\mathbf{I}\right)\right]$$

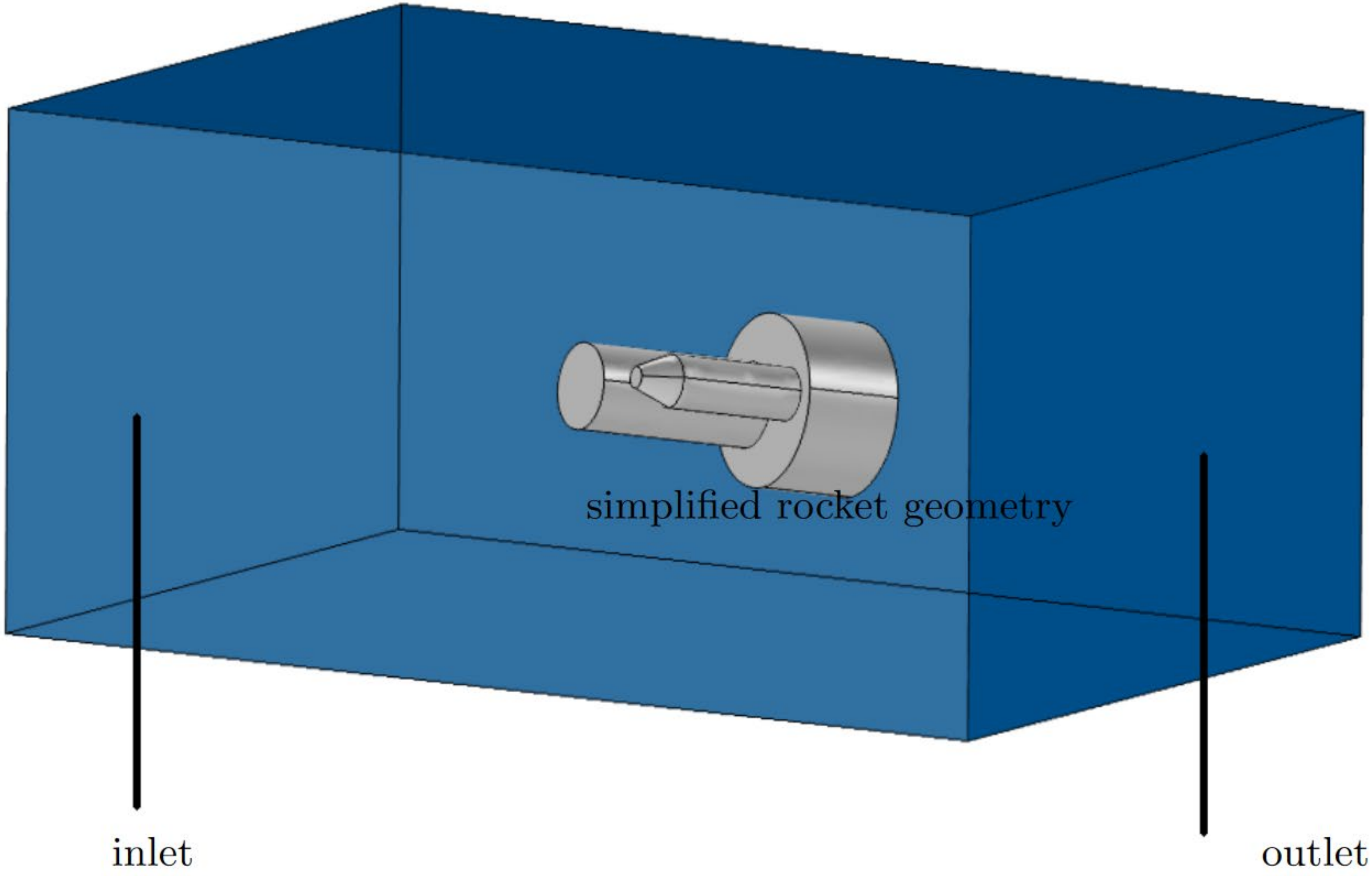
ρ: density
$\vec{u}$: velocity
p: pressure

μ: dynamic viscosity
$\vec{f}$: body force
e: internal energy

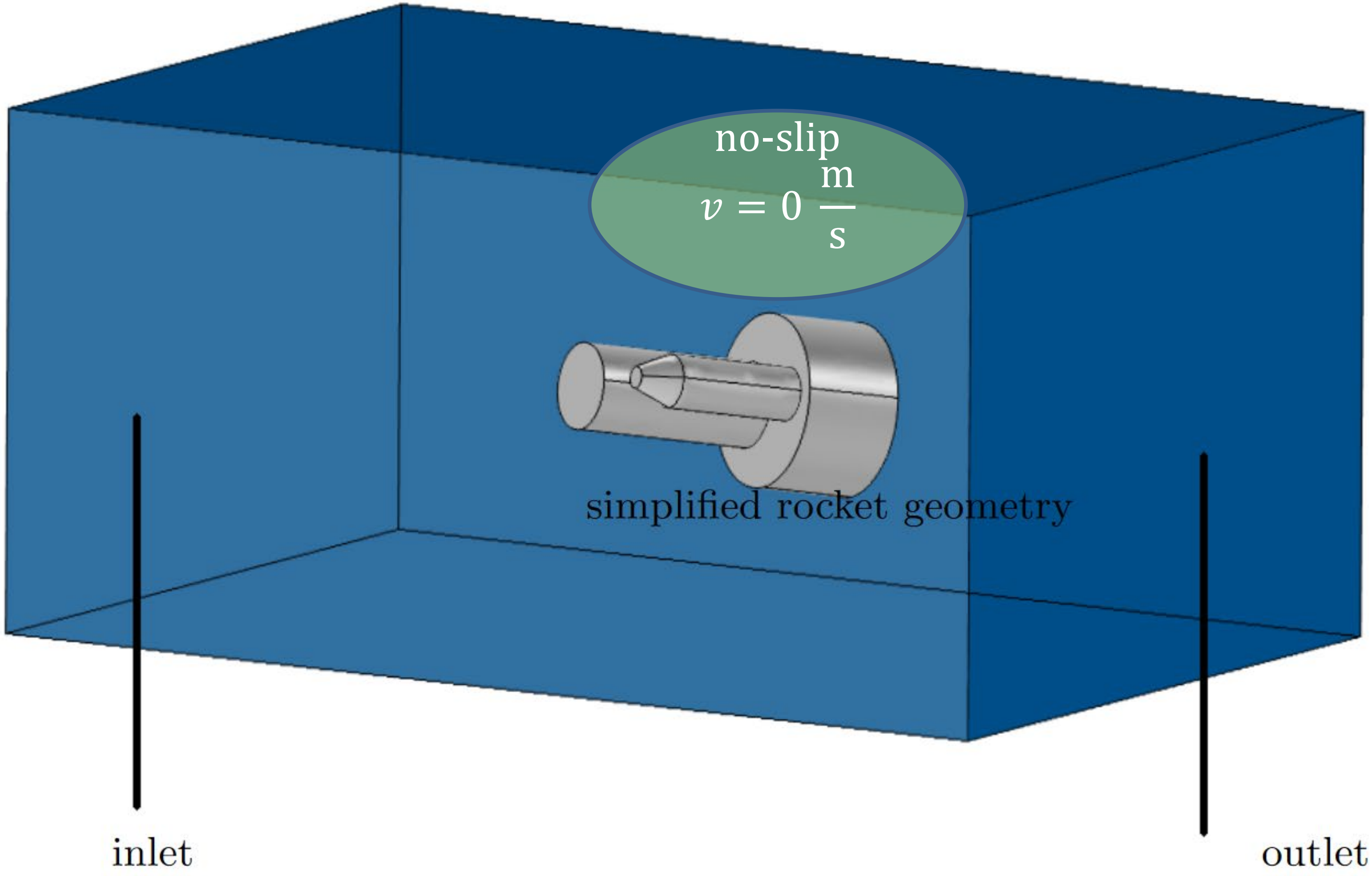
c_v: specific heat capacity
T: temperature
R_s: specific gas constant

$\vec{q} = k\nabla T$: heat flow vector
k: thermal conductivity

Simulation geometry



Simulation geometry



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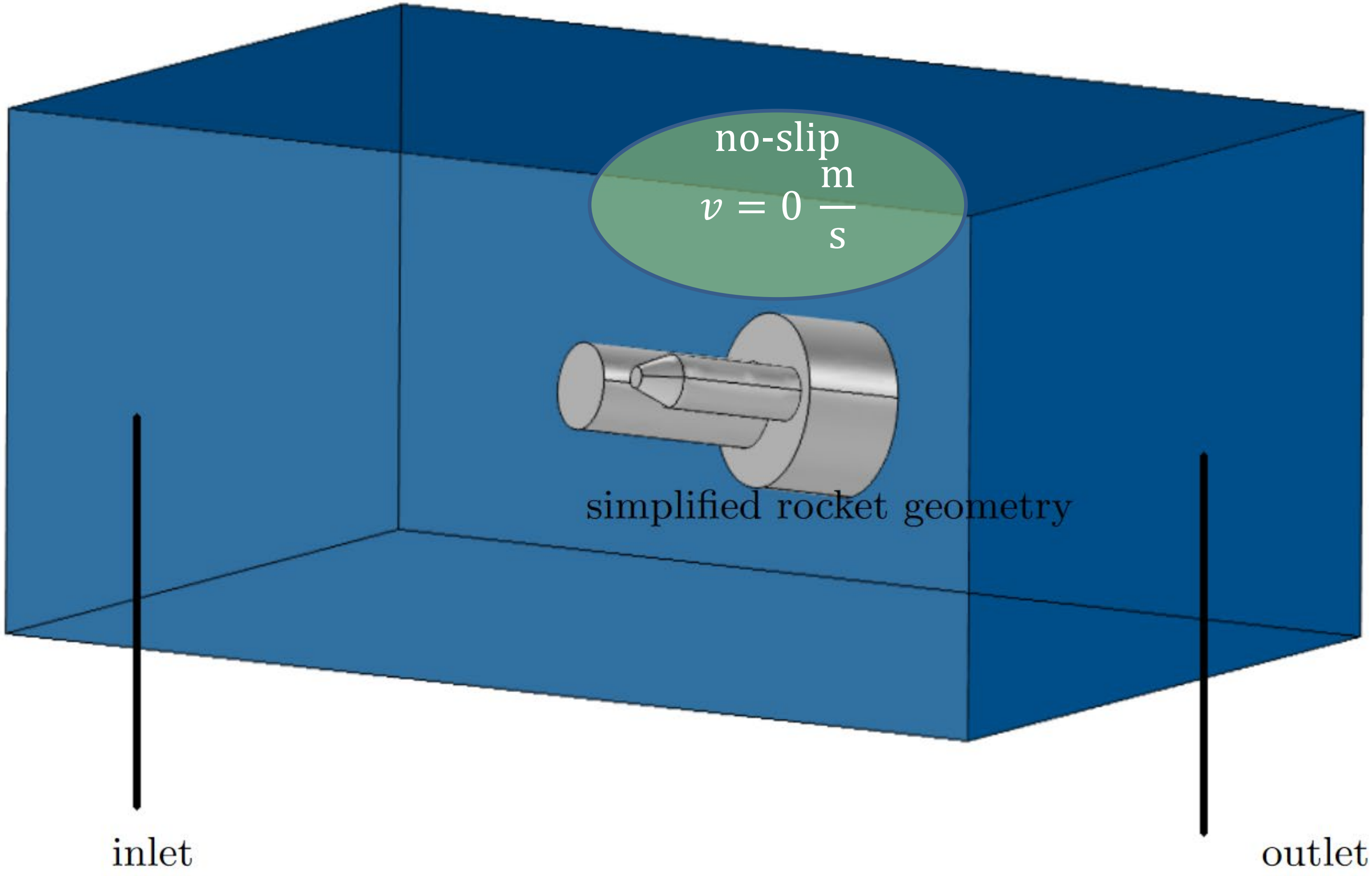
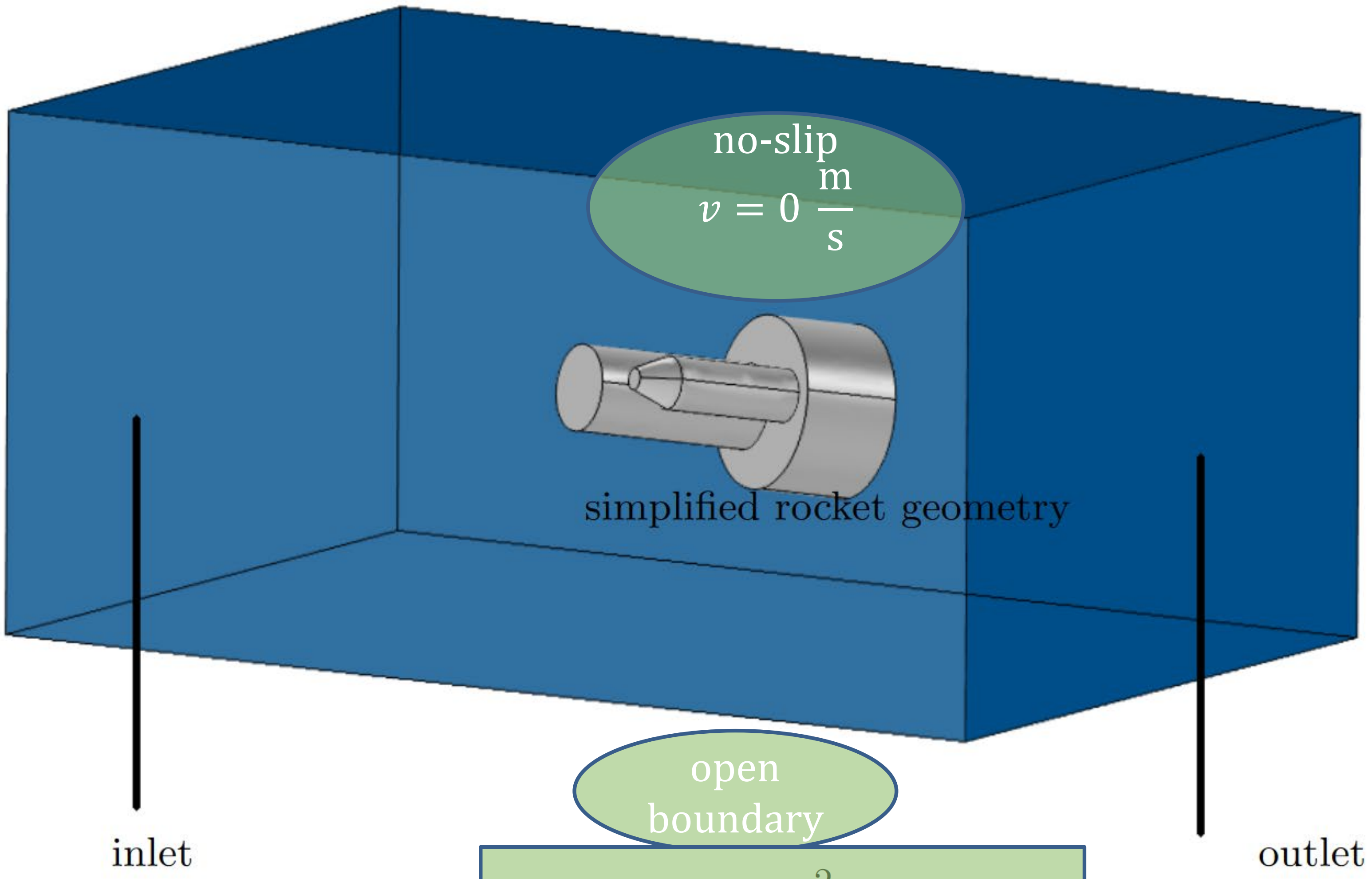


no-slip
$v = 0\ \frac{\text{m}}{\text{s}}$

$300\ \frac{\text{m}}{\text{s}}$

$400\ \frac{\text{m}}{\text{s}}$

Simulation geometry



$300 \, \frac{\mathrm{m}}{\mathrm{s}}$

$400 \, \frac{\mathrm{m}}{\mathrm{s}}$

no-slip
$v = 0 \, \frac{\mathrm{m}}{\mathrm{s}}$

open boundary

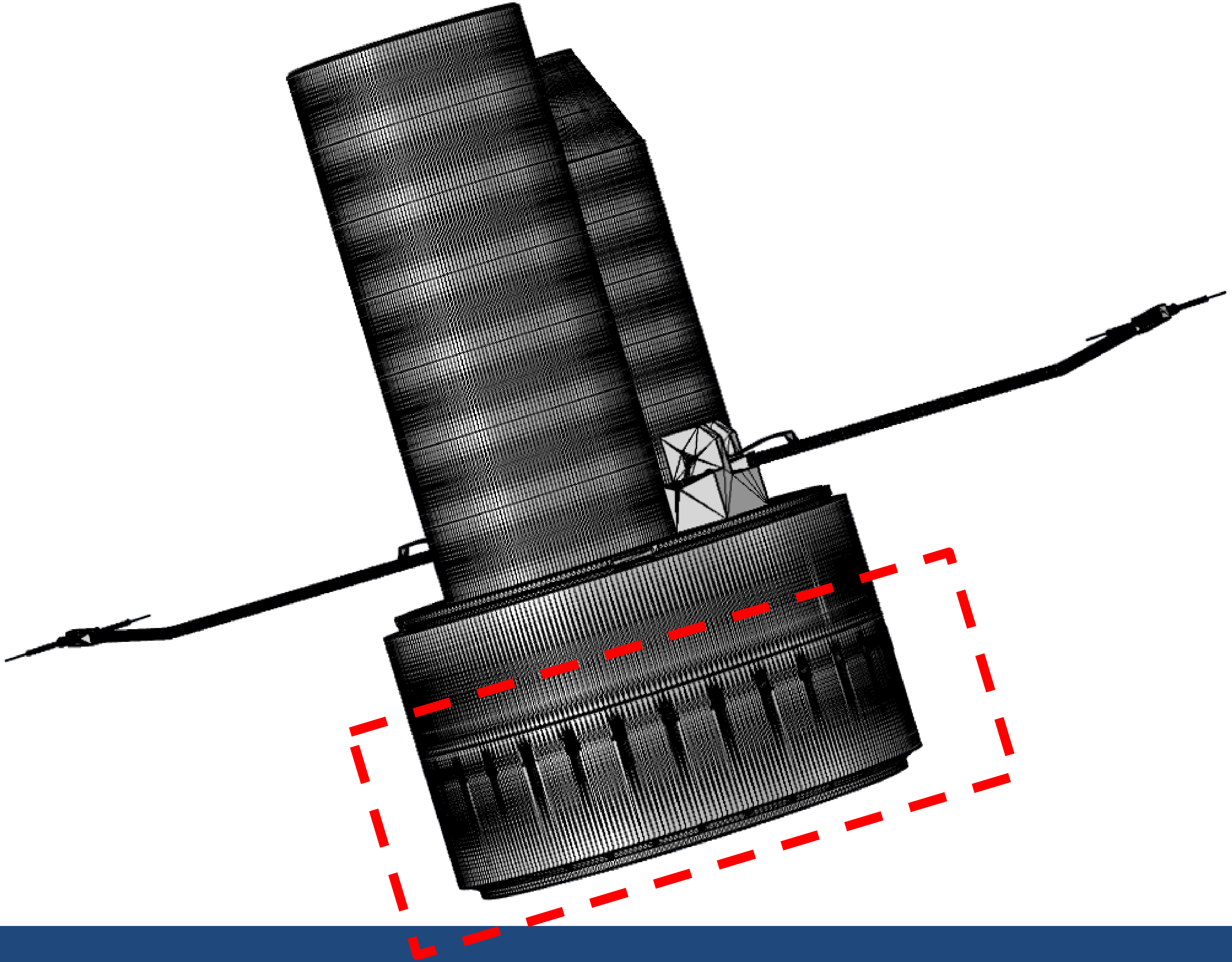
$$(\mu(\nabla\vec{u} + (\nabla\vec{u})^T) - \frac{2}{3}\mu(\nabla\vec{u})\mathbf{I})\vec{n} = 0$$

Computing data of our COMSOL model

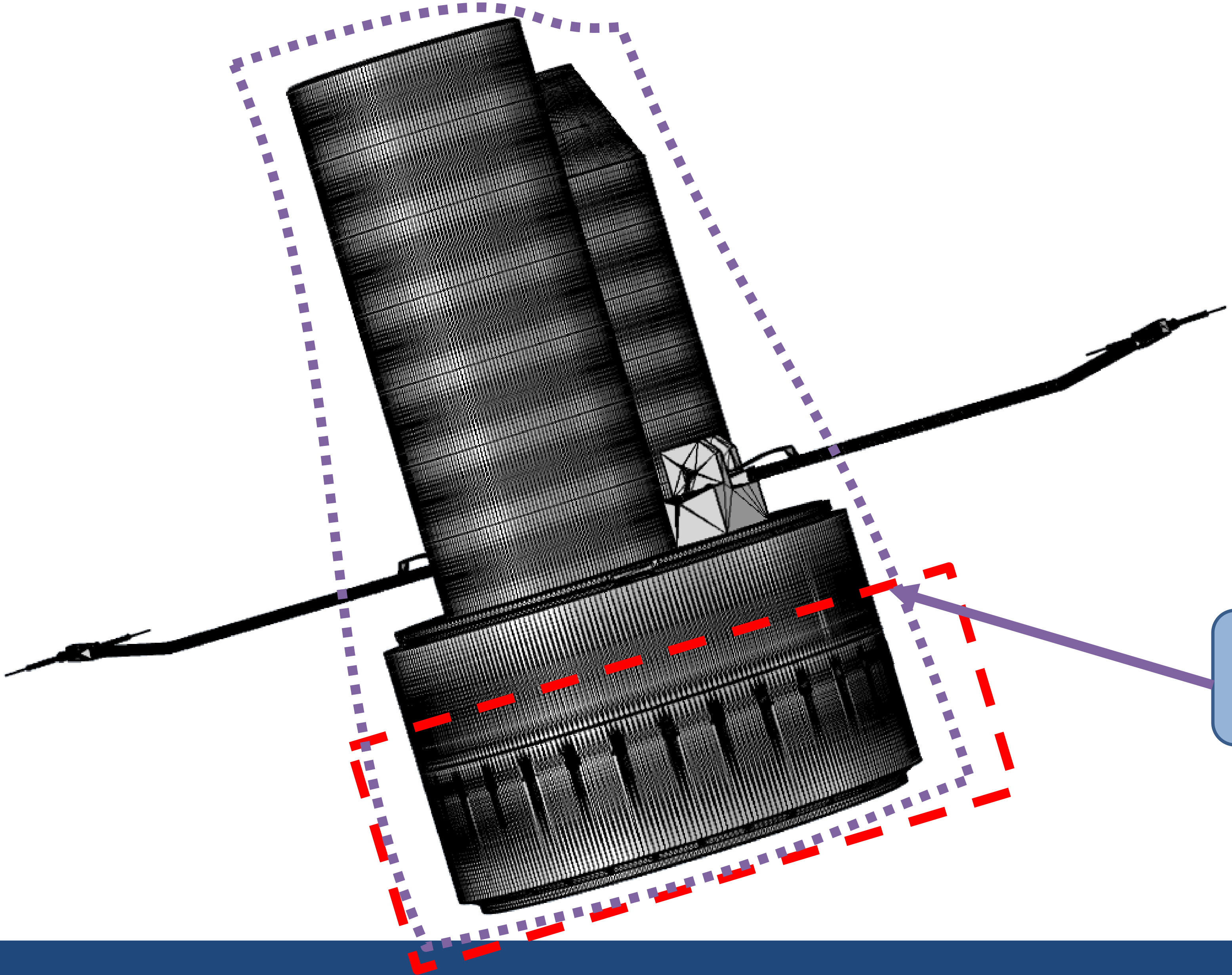
- Combination of tetrahedral, pyramidal, prismatic mesh elements
- ~4 million mesh elements
- Smallest element sizes: ~ 1 mm
- Time discretization by BDF (backward differentiation formula) method
- Cluster computing:
  - 6 nodes
  - 70 cores
  - 396 GB RAM (working storage)
- Solving time up to 20 days

Simulation based development of the particle collector

Development of the particle collector

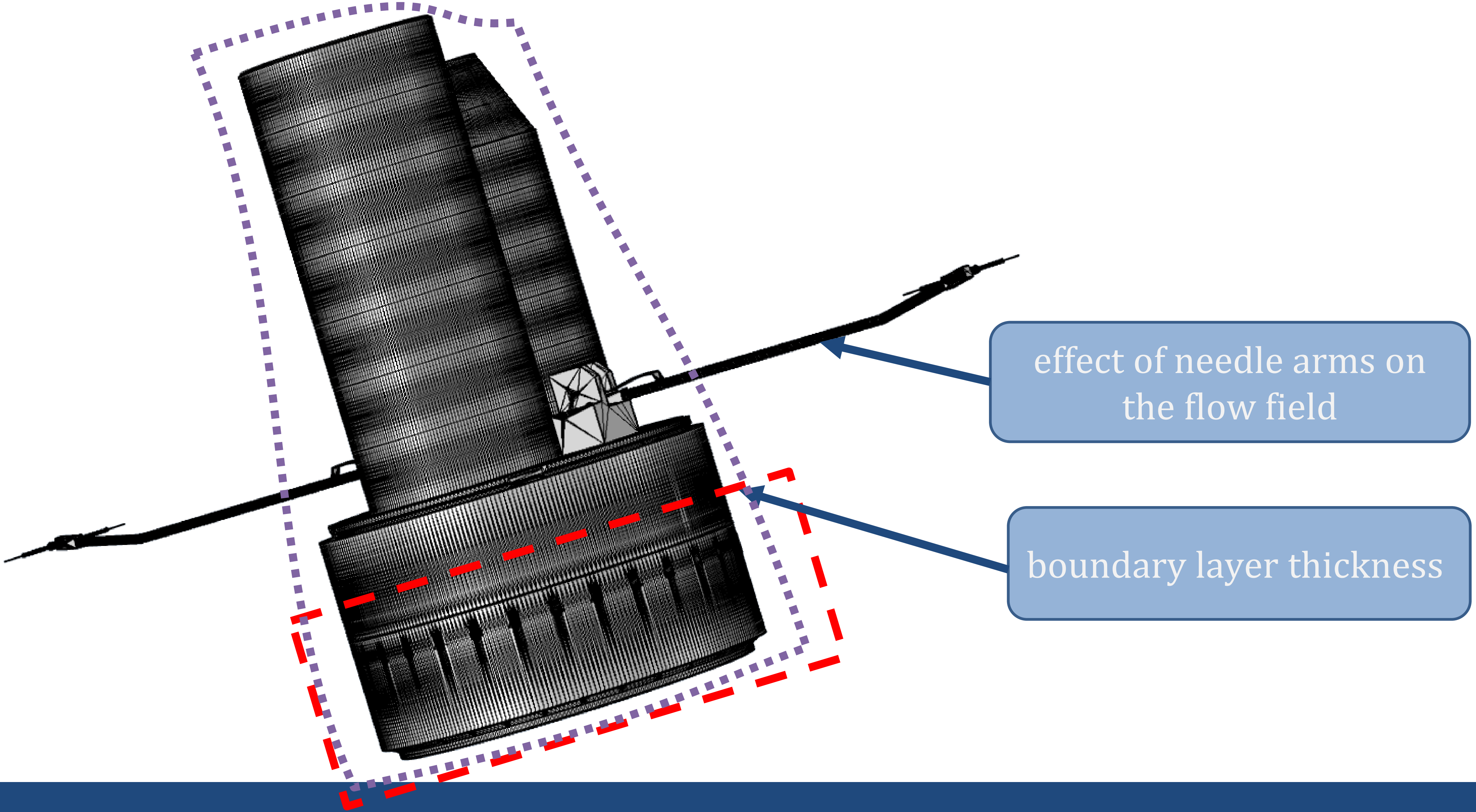


Development of the particle collector



boundary layer thickness

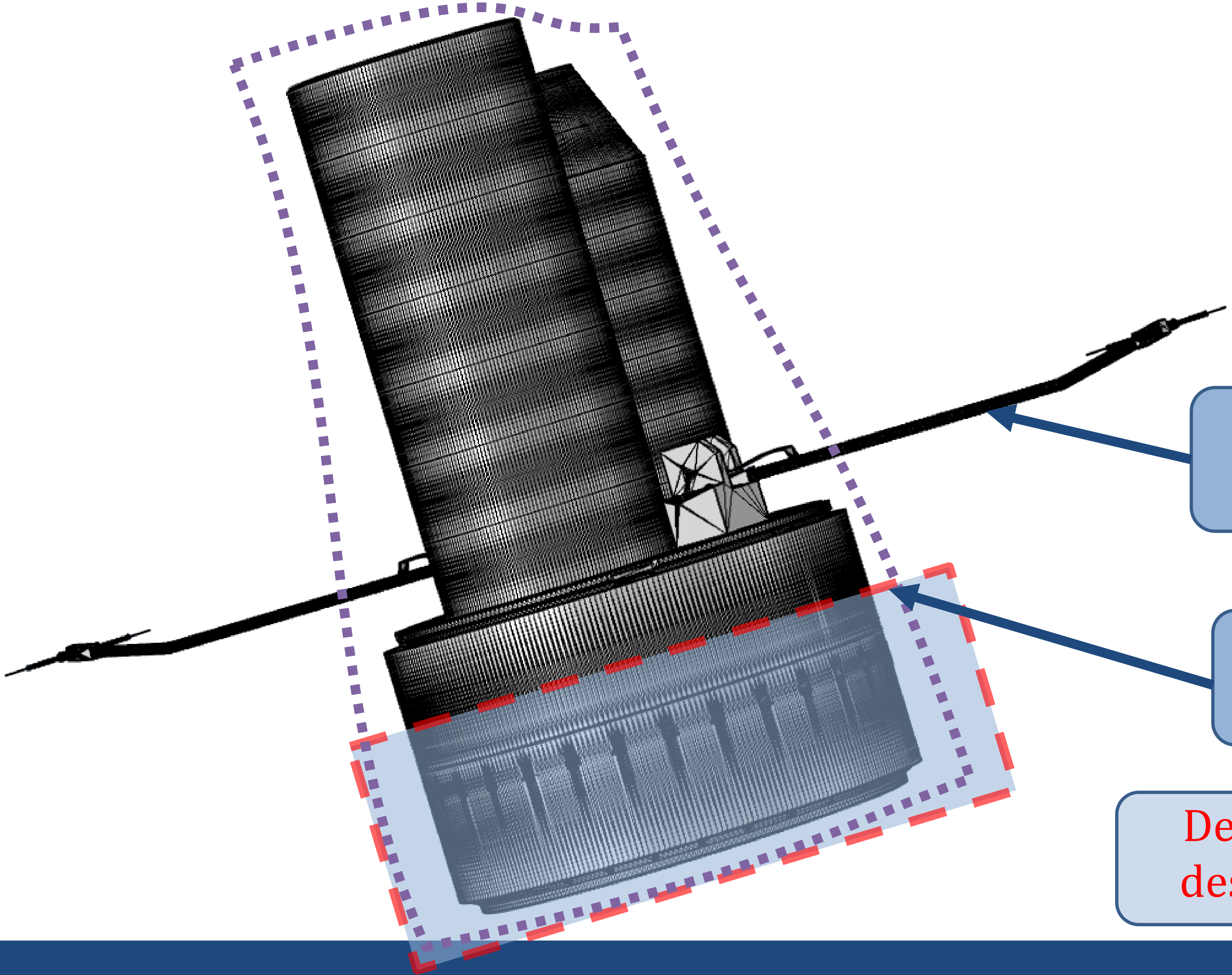
Development of the particle collector



effect of needle arms on the flow field

boundary layer thickness

Development of the particle collector

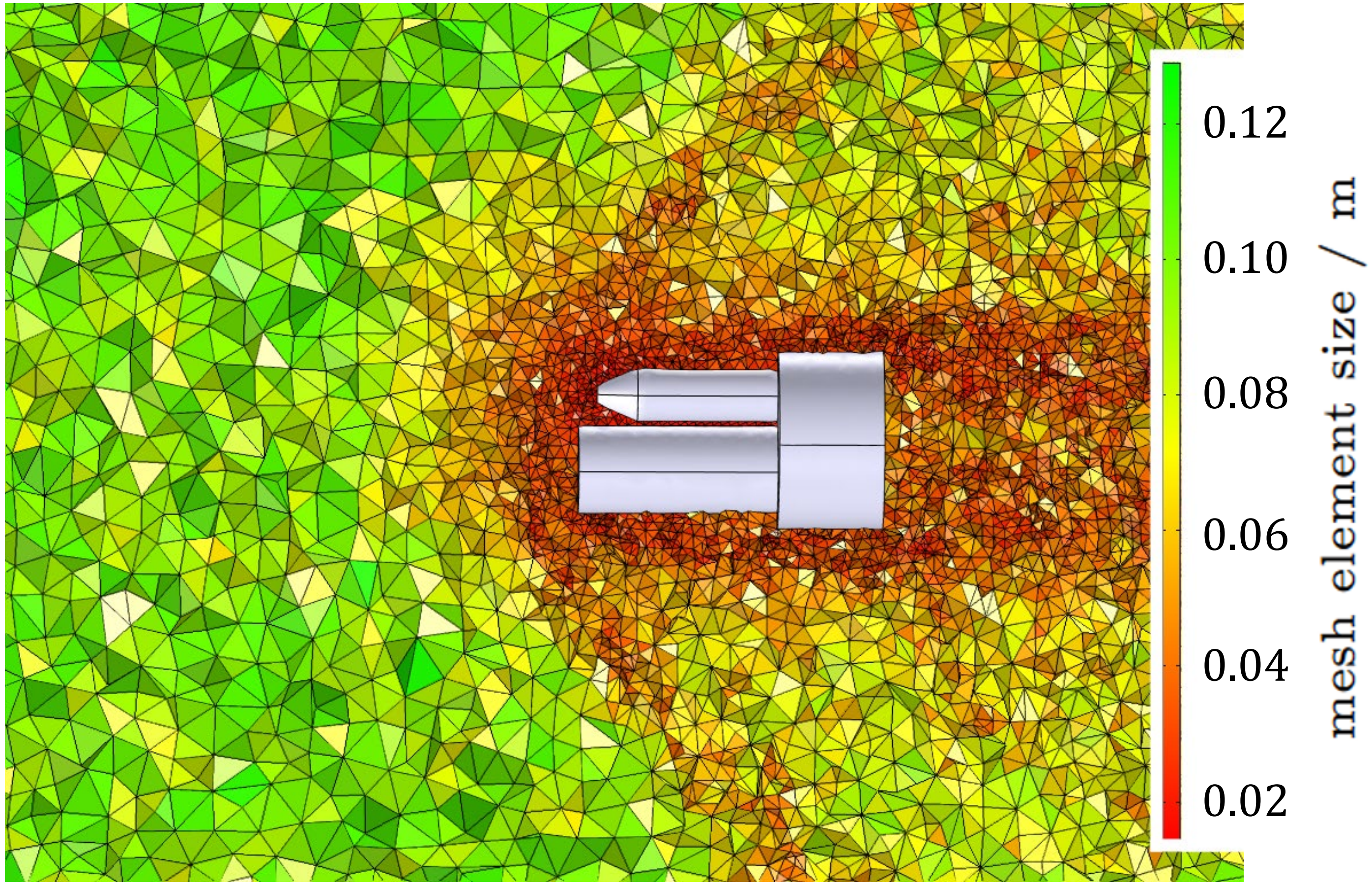


effect of needle arms on the flow field

boundary layer thickness

Determination of location and design of the particle collector

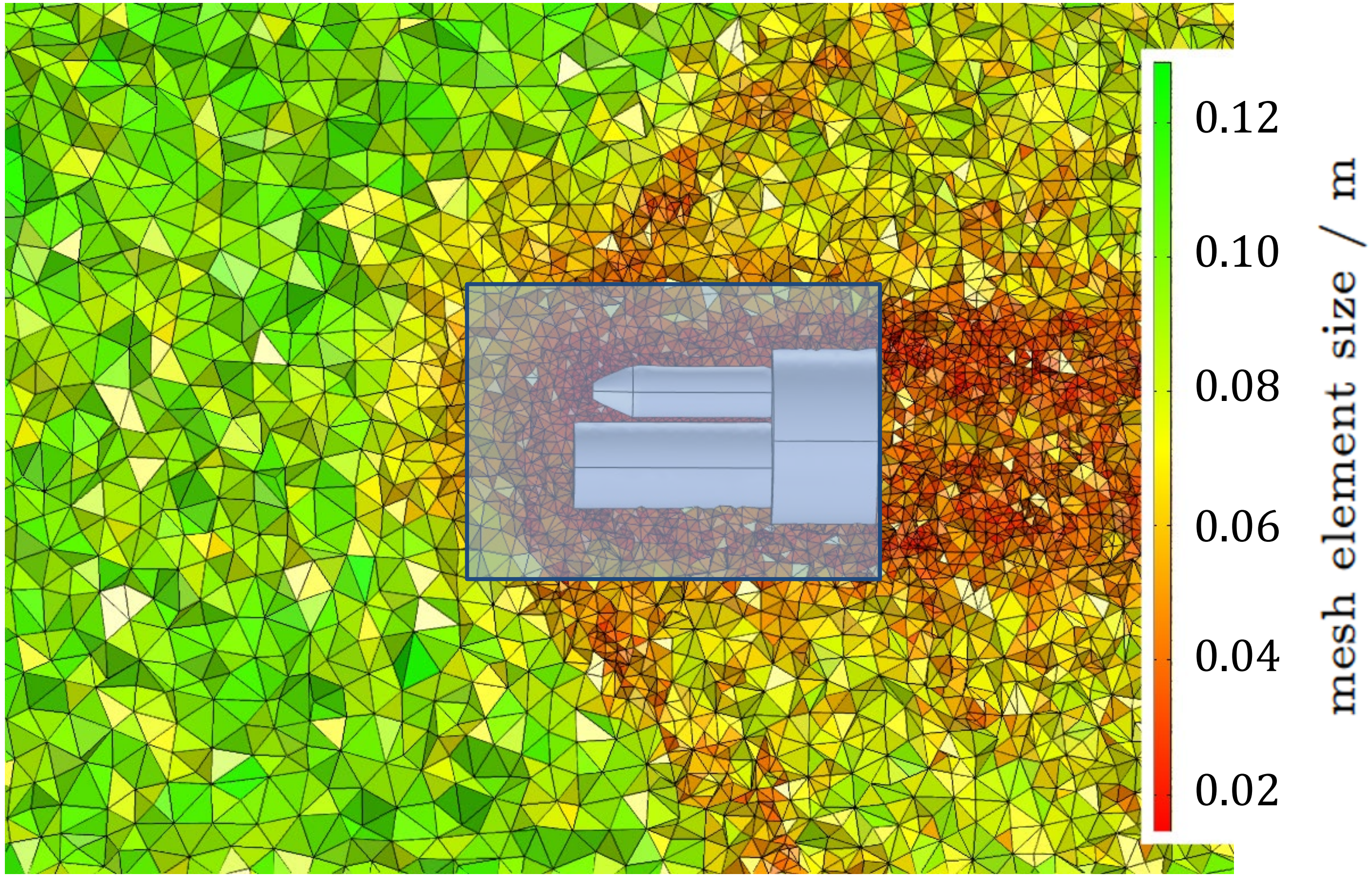
Generated 3D mesh for simulations



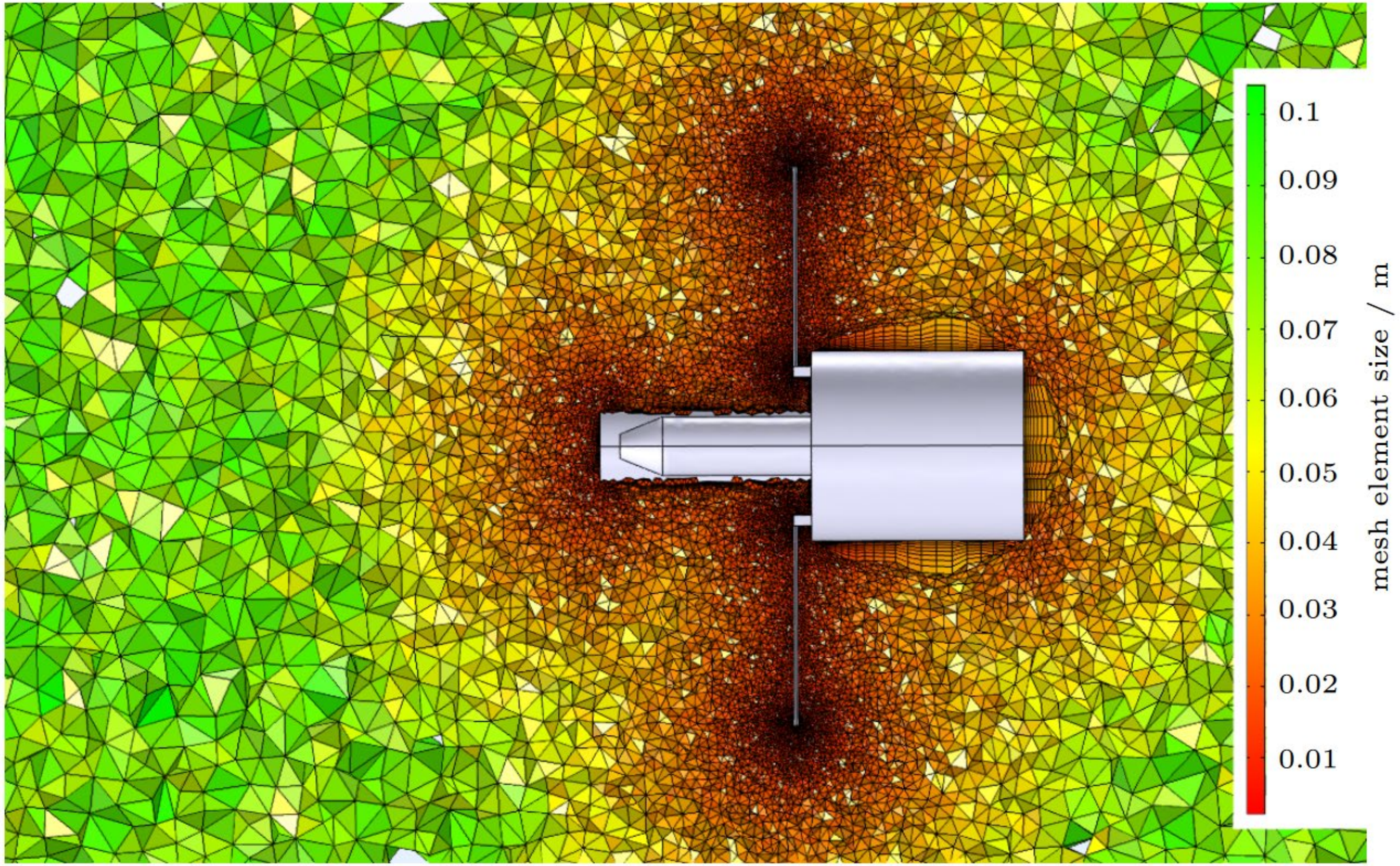
Mesh refinement by the error indicator:

$$||\nabla \vec{u}||$$

Generated 3D mesh for simulations

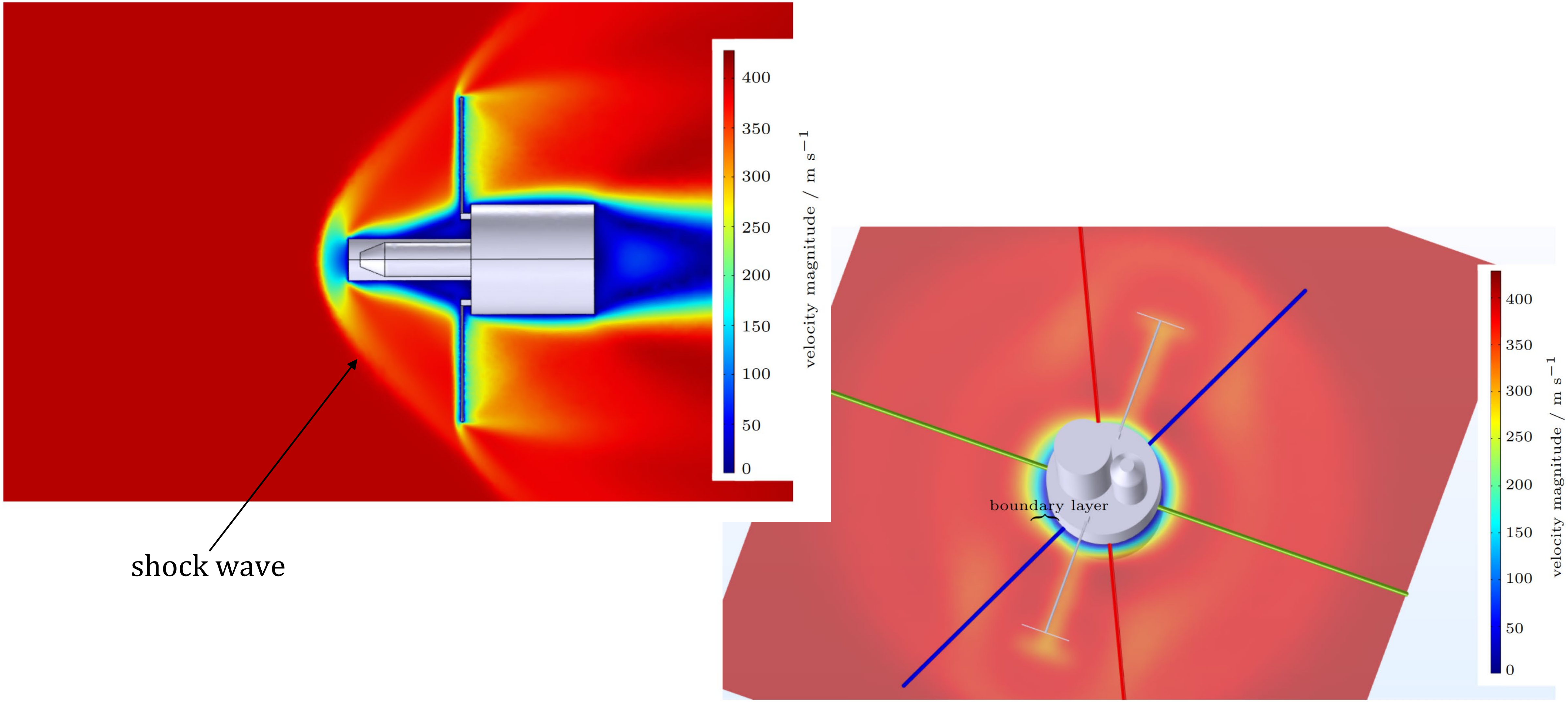


Generated 3D mesh for simulations



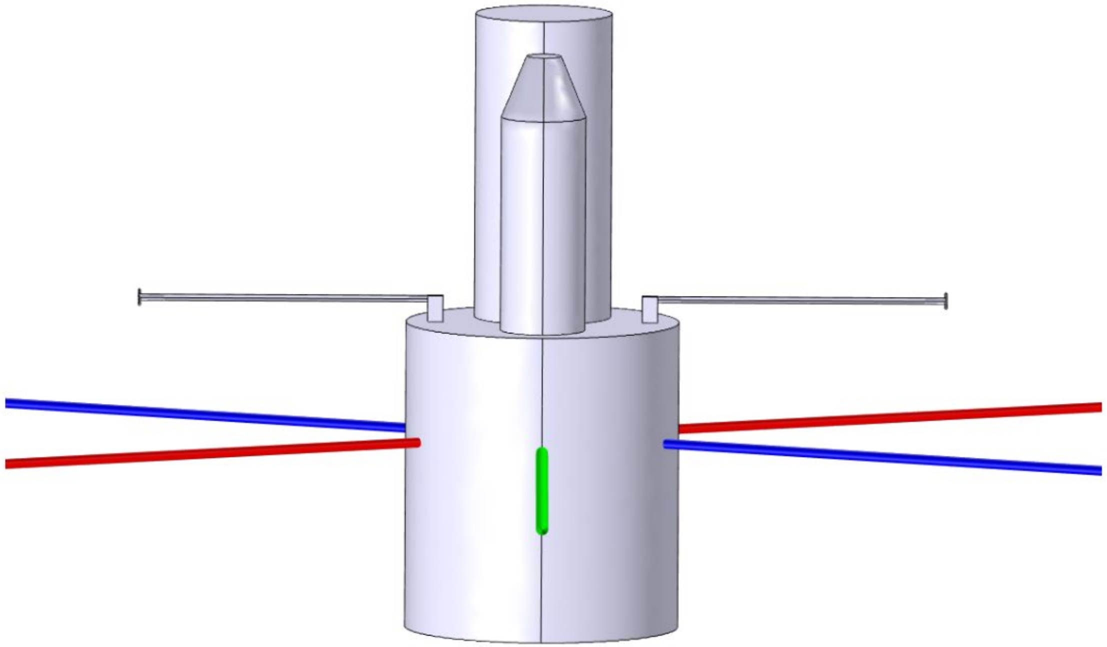
Mesh element size in m

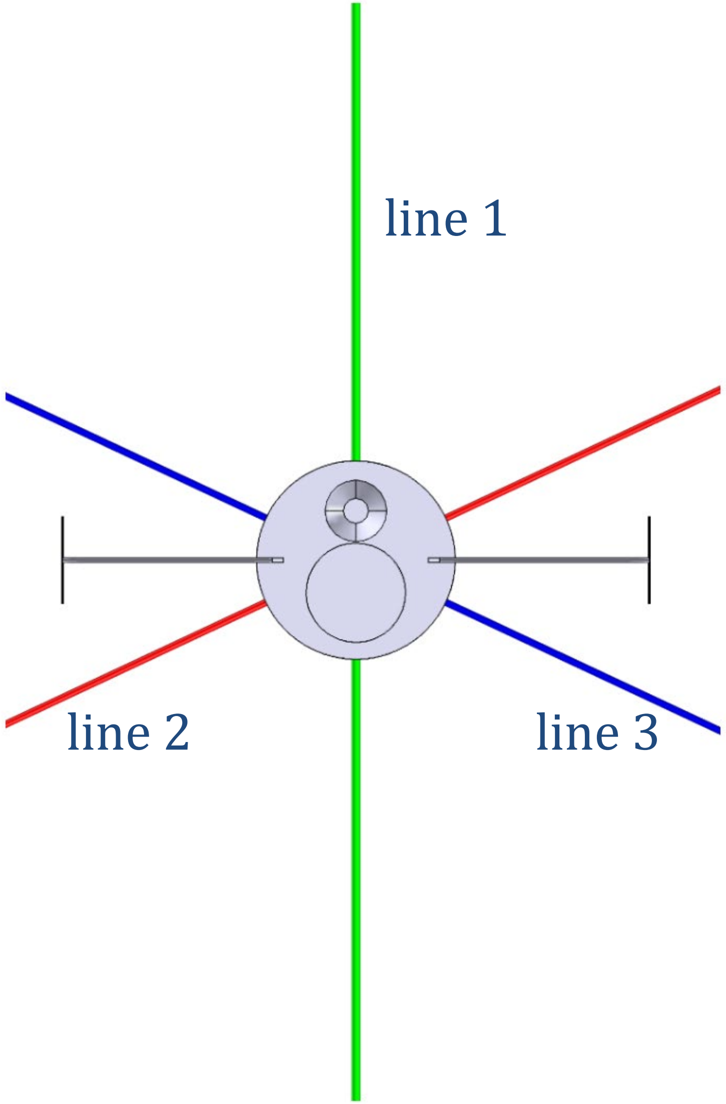
Flow field around the payload tip



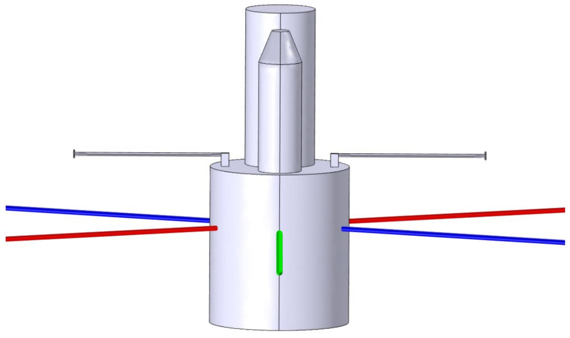
Velocity magnitude depicted on a cut plane perpendicular to the payload tip

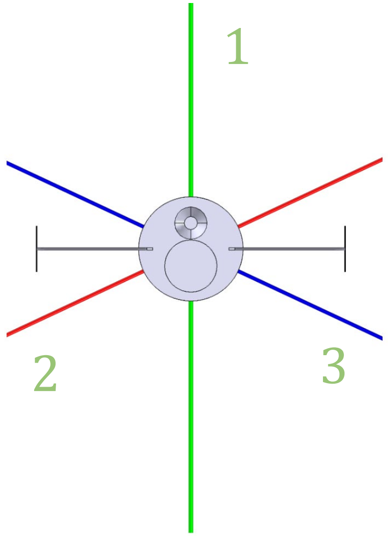
Analyzing velocity magnitudes along cut lines

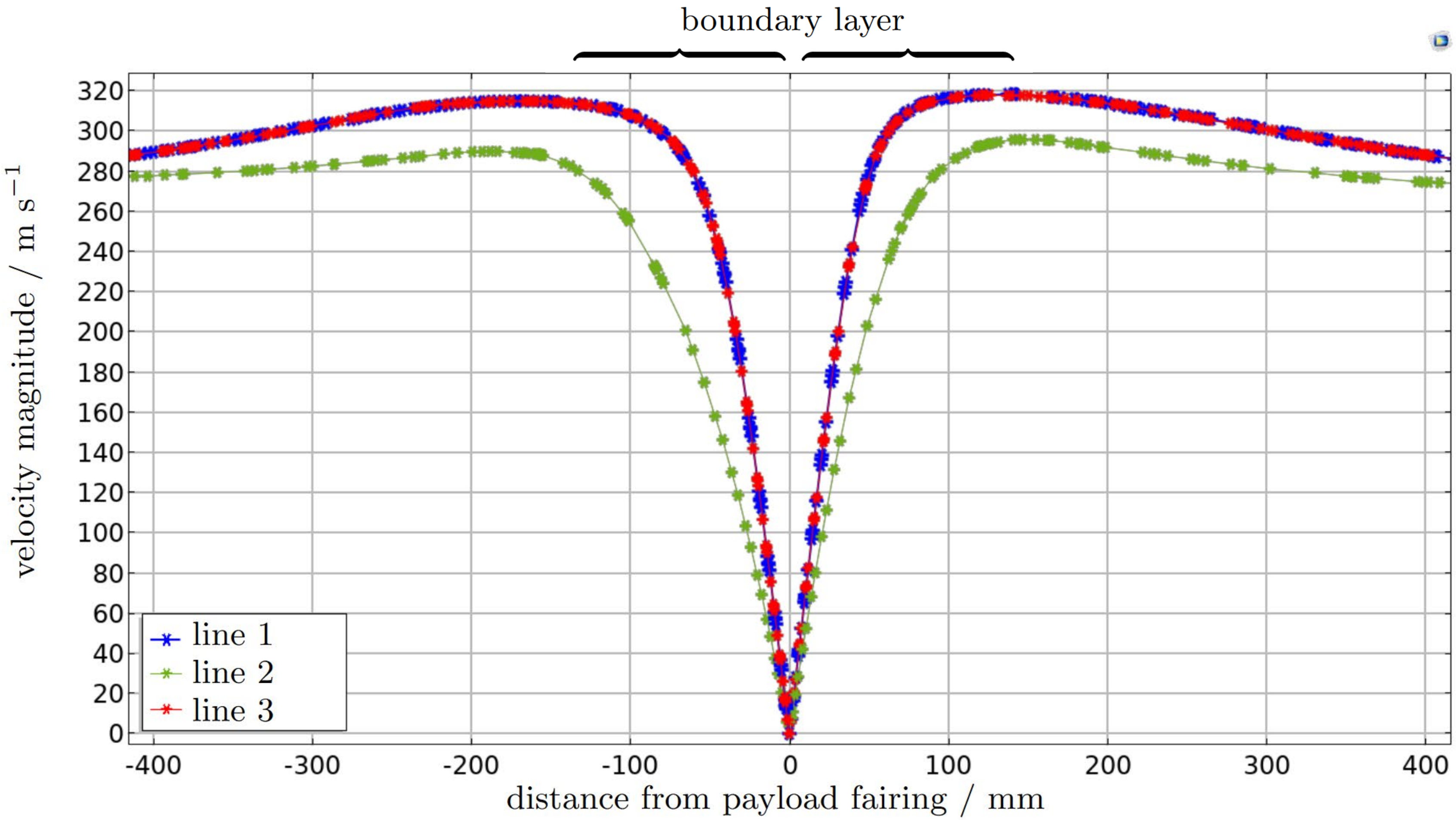




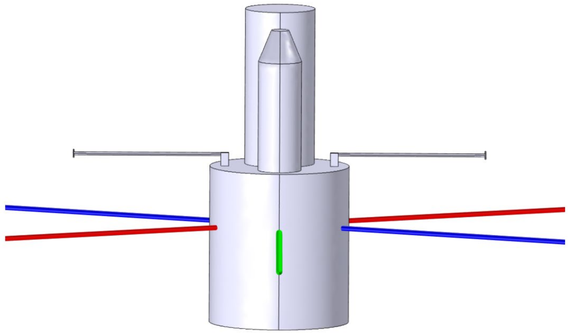
Analyzing velocity magnitudes along cut lines

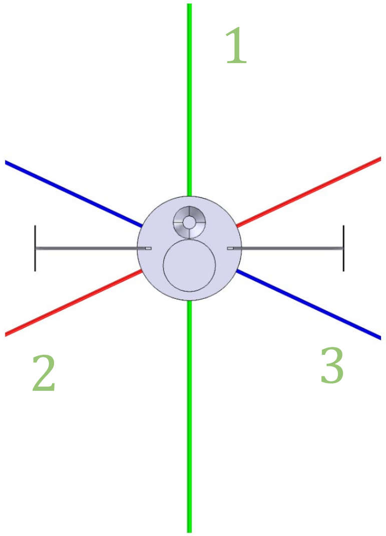


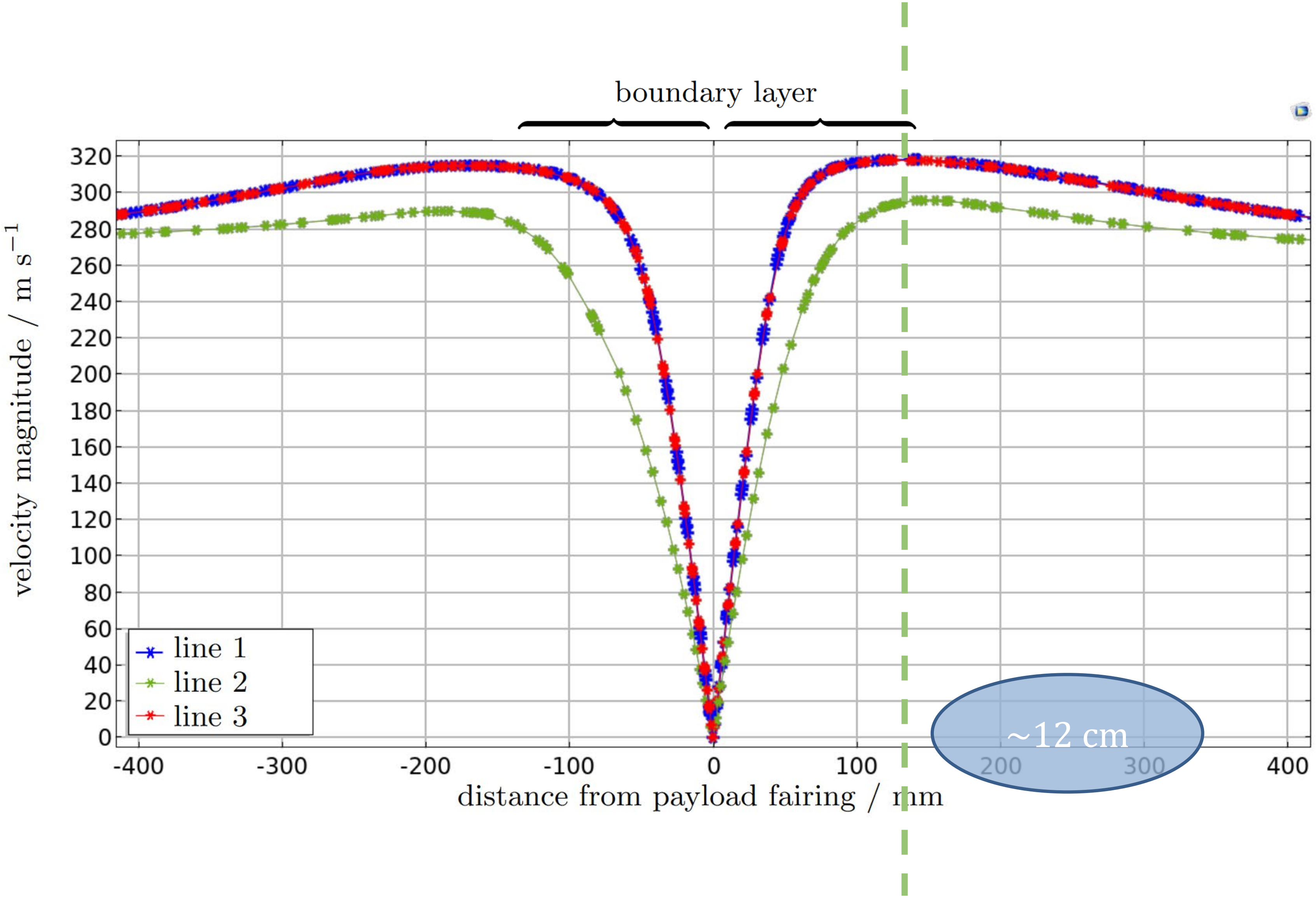




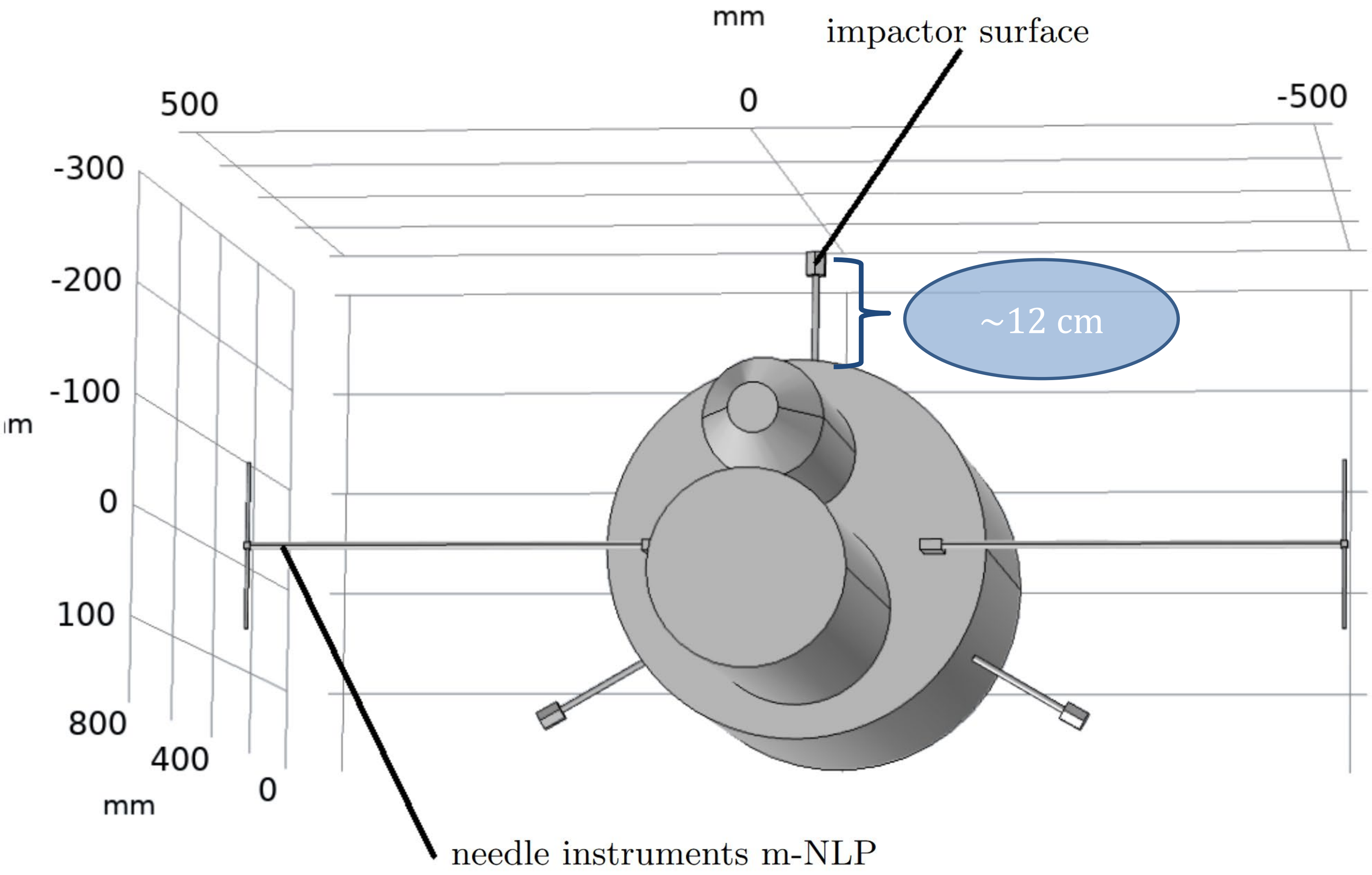
Analyzing velocity magnitudes along cut lines



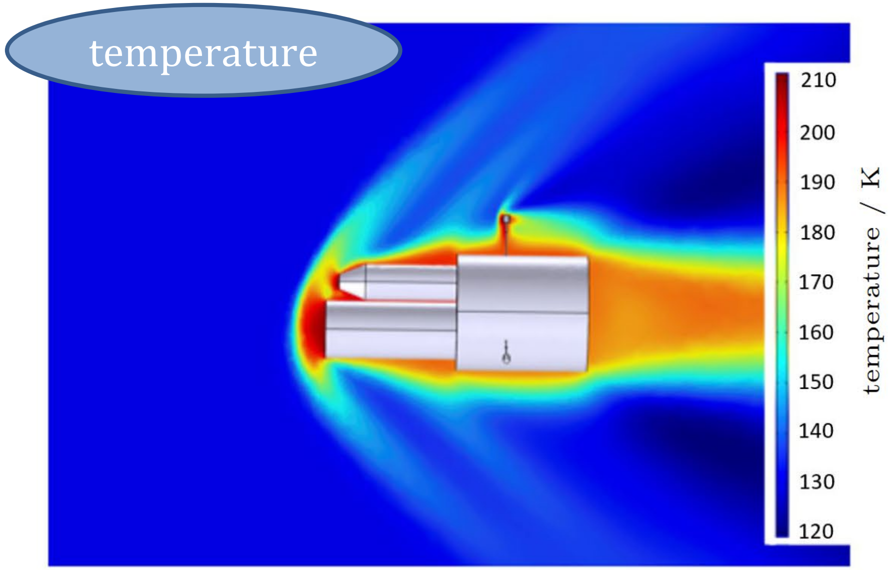


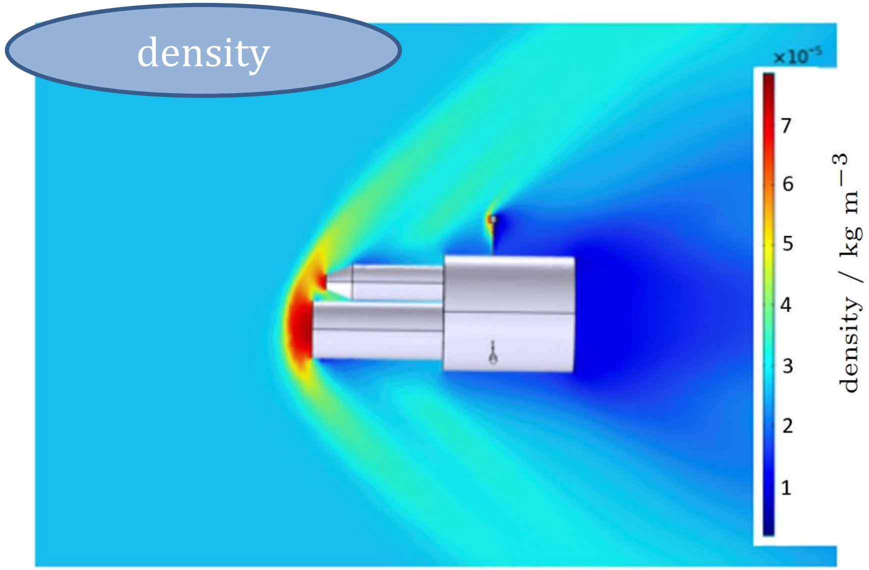


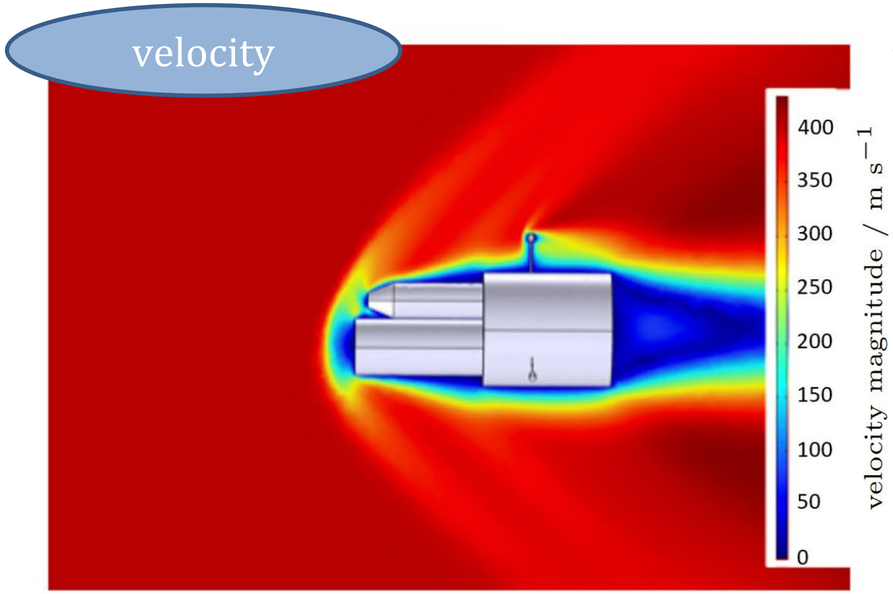
Final location and design of the samplers

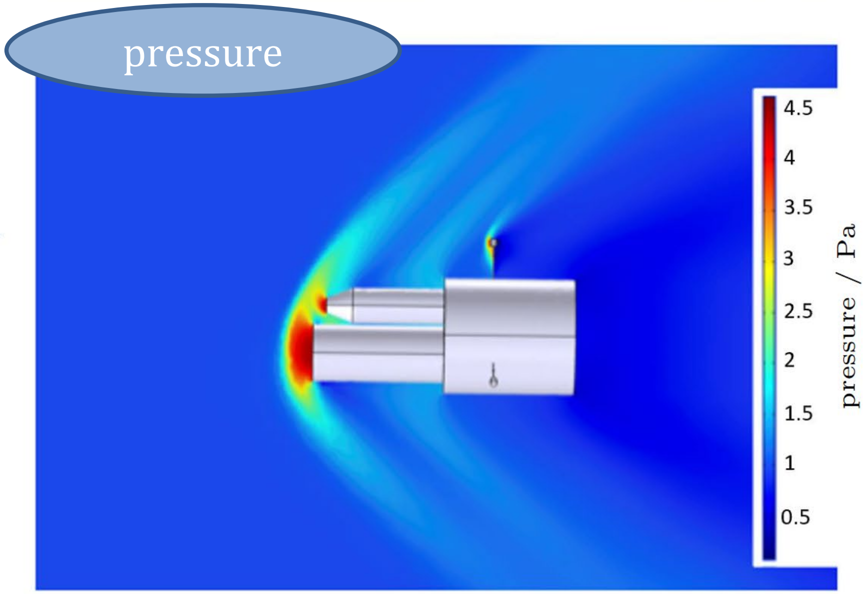


Flow field around the payload tip 0° angle of attack

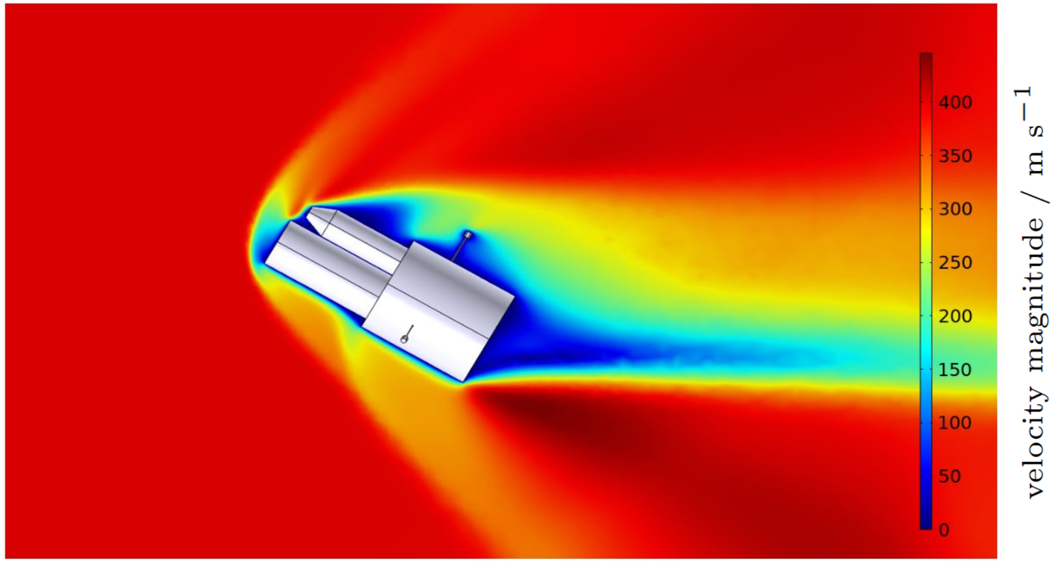

temperature

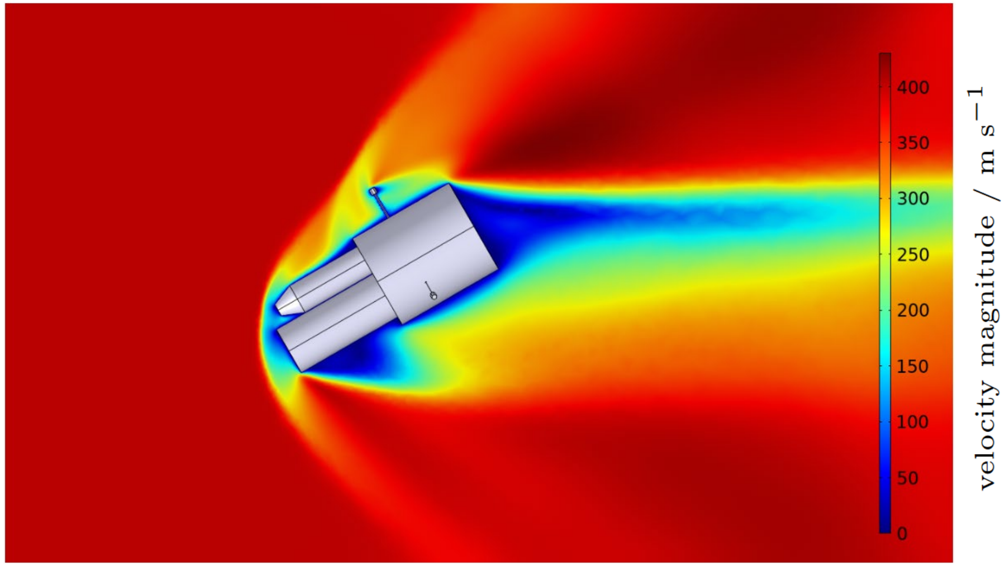

density


velocity


pressure

Flow field around the payload tip at ±30° angle of attack





Mathematical and numerical model of particle simulations

Particle data

Particle volume fraction

$$\Phi_p = \frac{nV_p}{V} \leq 10^{-6}$$ ✓

one-way coupling

Knudsen number

$$Kn_p = \frac{2\lambda}{d_p} > 1$$

Cunningham slip corrector

n: number of particles
V_p: particle volume
V: fluid volume

Kn_p: Knudsen number
λ : mean free path
d_p: particle diameter

Estimation of particle forces

forces	mathematical expressions	estimated magnitude of forces / N
Brownian force	$\vec{F}_{Brown} = \vec{\zeta}\sqrt{\frac{6\ \pi\ \mu\ \mathrm{k_B}\ T\ d_p}{\Delta t\ C_c}}$	$\sim 10^{-18}$
Stokes drag force	$\vec{F}_D = 3\ \pi\ \mu_f\ d_p\ \vec{u}_r\ C_c^{-1}$	$\sim 10^{-18}$
Saffman force	$\vec{F}_S = 1.615\ d_p^2\ \vec{L}_f\ \sqrt{\rho_f \mu_f \frac{1}{\lvert\nabla\times\vec{u}_r\rvert}}$	$\sim 10^{-22}$
gravitational force	$\vec{F}_{G,tot} = m_p\ \frac{\rho_p - \rho_f}{\rho_p}\ \vec{g}$	$\sim 10^{-23}$
added mass force	$\vec{F}_{am} = m_f\ c_{am}\ \frac{\mathrm{d}(\vec{u}_f - \vec{v}_p)}{\mathrm{d}t}$	$\sim 10^{-25}$
pressure gradient force	$\vec{F}_p = -\frac{m_p}{\rho_p}\nabla p$	$\sim 10^{-28}$

Estimation of particle forces

forces	mathematical expressions	estimated magnitude of forces / N
Brownian force	$\vec{F}_{Brown} = \vec{\zeta}\sqrt{\frac{6\ \pi\ \mu\ \mathrm{k_B}\ T\ d_p}{\Delta t\ C_c}}$	$\sim 10^{-18}$
Stokes drag force	$\vec{F}_D = 3\ \pi\ \mu_f\ d_p\ \vec{u}_r\ C_c^{-1}$	$\sim 10^{-18}$

$$\boxed{\vec{F} = m \cdot \vec{a}}$$

$$\boxed{m_p \frac{d^2\,\vec{x}}{dt^2} = \vec{\xi}\sqrt{\frac{6\pi\mu \mathrm{k}_B T d_p}{\Delta t C_c}} + \frac{3\pi\mu_f d_p \vec{u}_r}{C_c}}$$

Brownian force

Stokes drag force

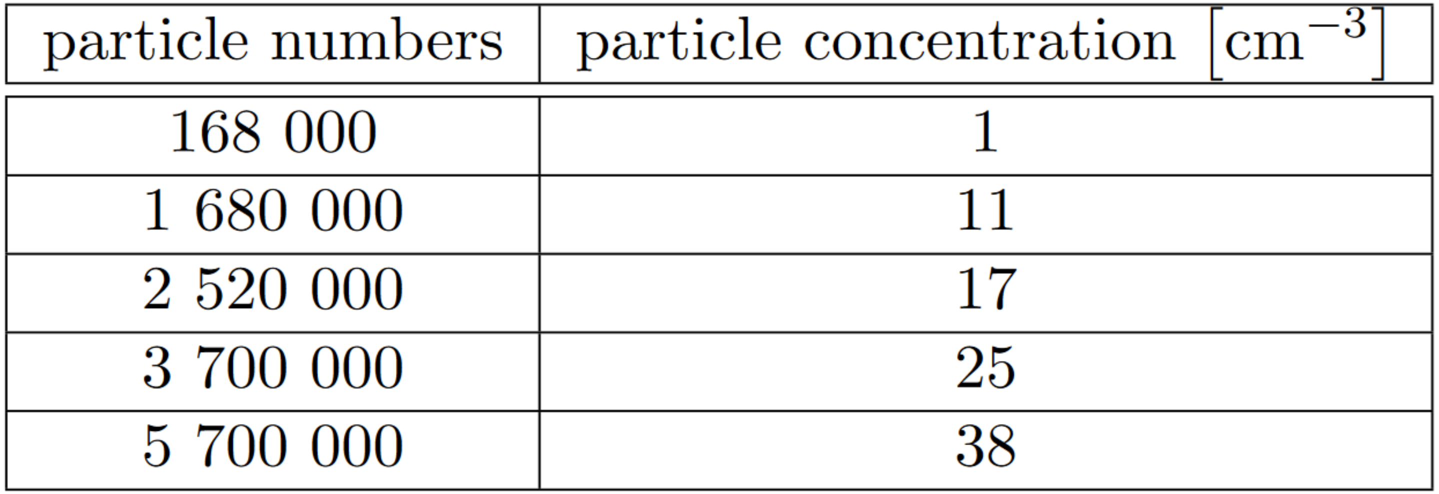
ρ: density
$\vec{x}$: particle position
$\mathrm{k_B}$: Bolzmann constant

T: temperature
d_p: particle diameter
$\mathrm{C_C}$: Cunningham slip corrector

$\vec{\zeta}$: vector of random numbers
μ_f: fluid viscosity
$\mathrm{C_C}$: Cunningham slip corrector

$\overrightarrow{u_r}$: relative particle velocity

Particle calculations

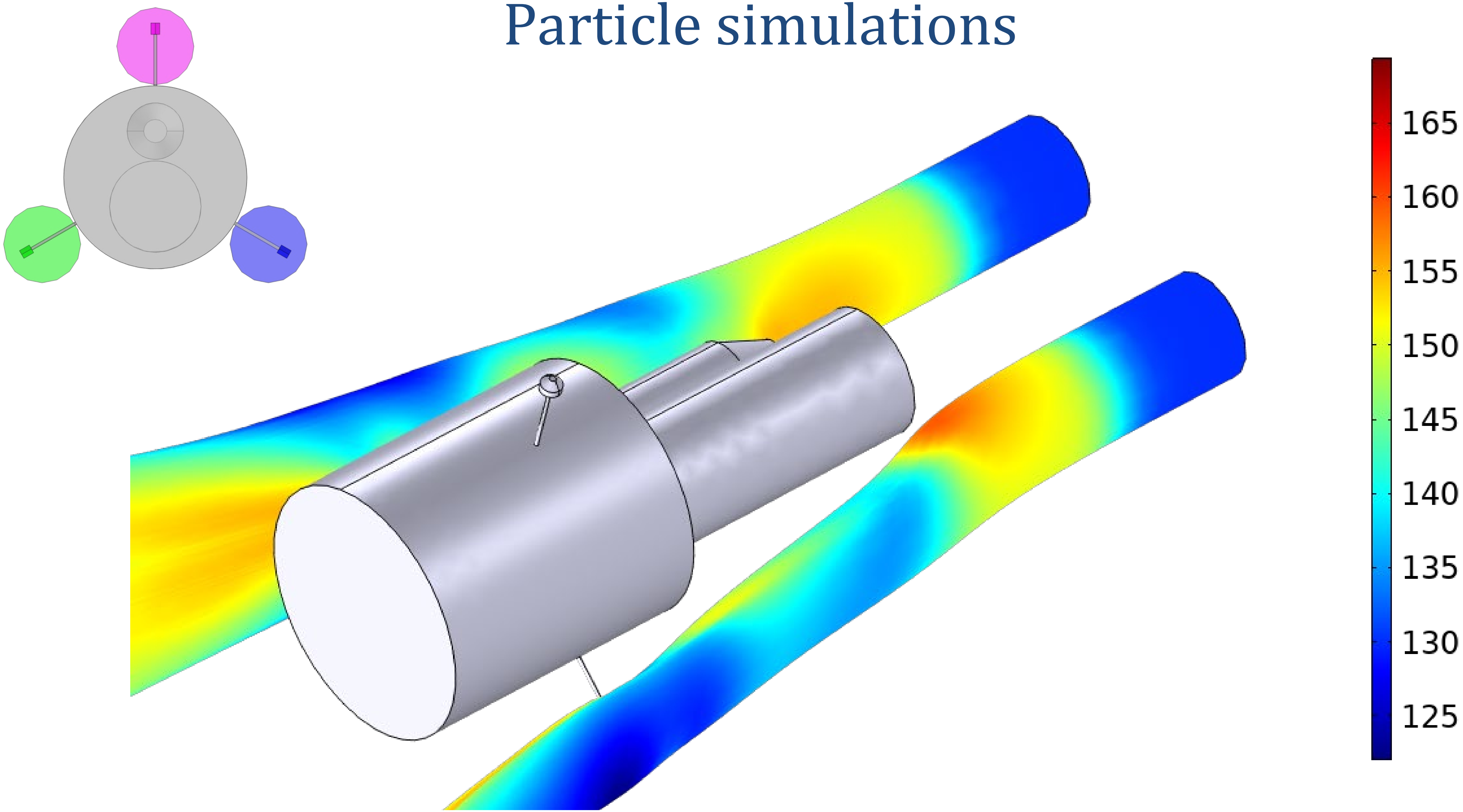


particle numbers	particle concentration $\left[\mathrm{cm}^{-3}\right]$
168 000	1
1 680 000	11
2 520 000	17
3 700 000	25
5 700 000	38

- Comsol implemented generalized α-method
- Parametric sweep
- Total solving time: $\sim$3 days

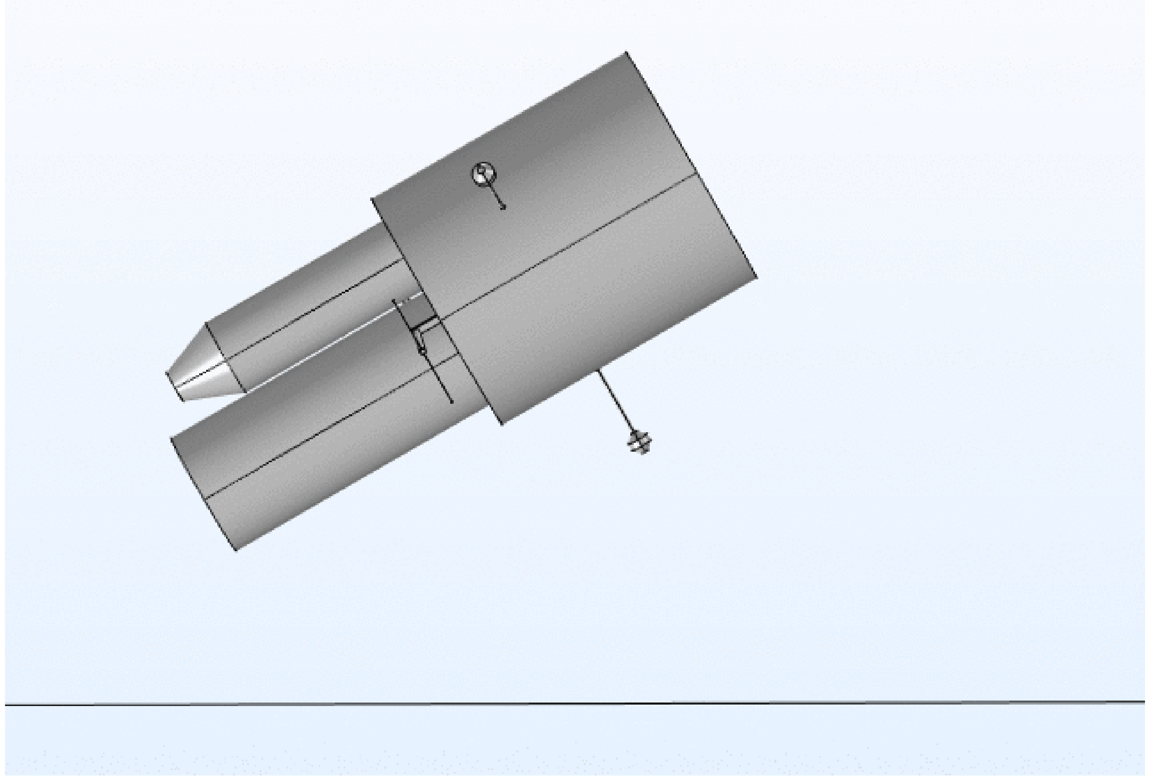
Simulation setup and results of particle calculations

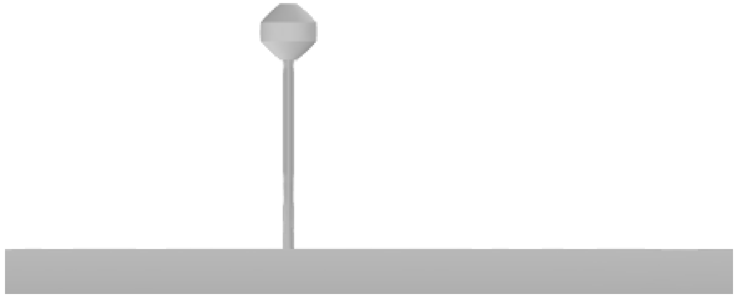
Particle simulations



Particle simulation

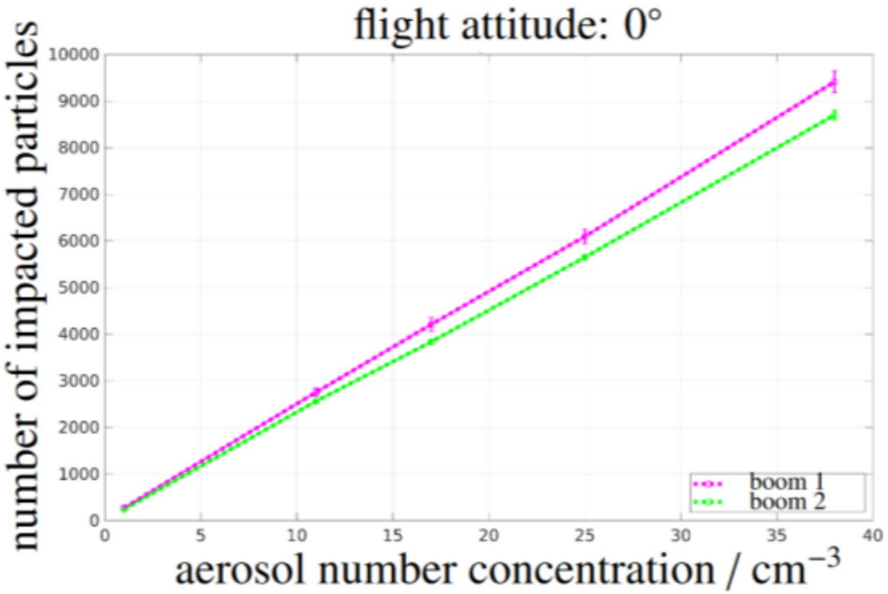
Particle velocity / $\frac{\mathrm{m}}{\mathrm{s}}$

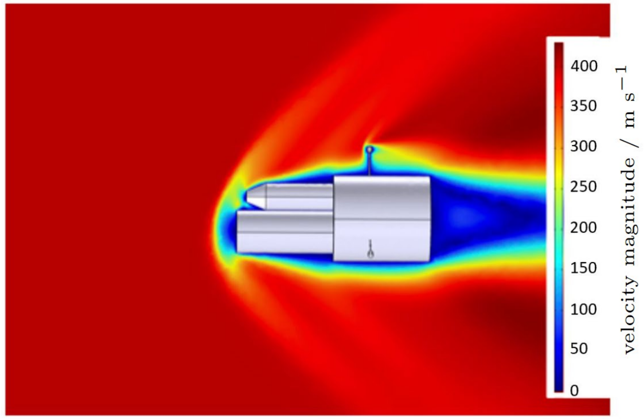




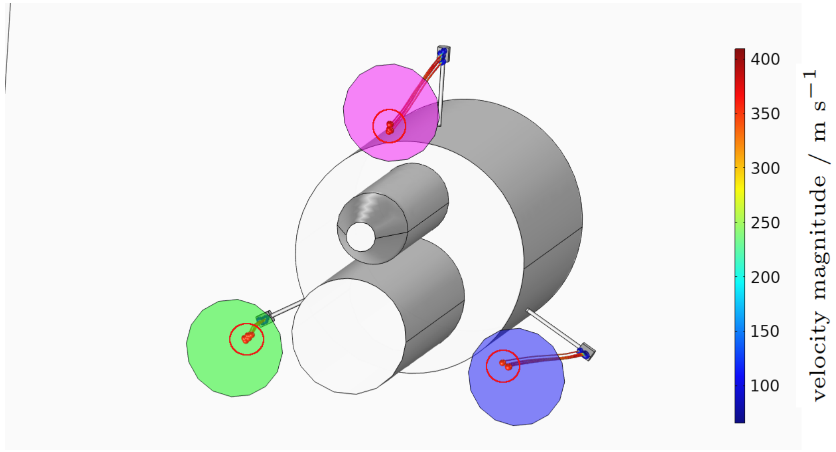
Particle velocity / $\frac{\mathrm{m}}{\mathrm{s}}$

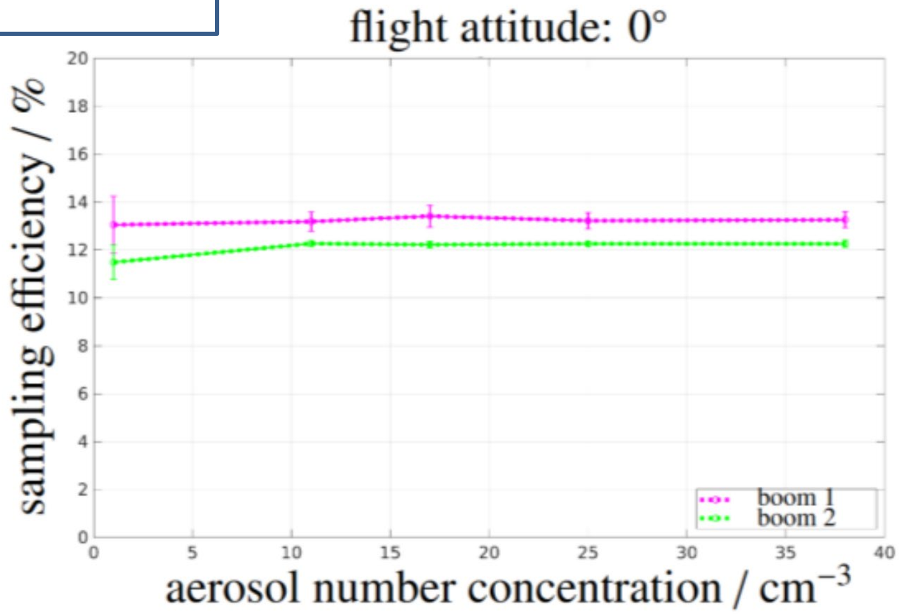
Particle simulation results





$$\text{efficiency} = \frac{C_{impacted}}{C_{released}}$$





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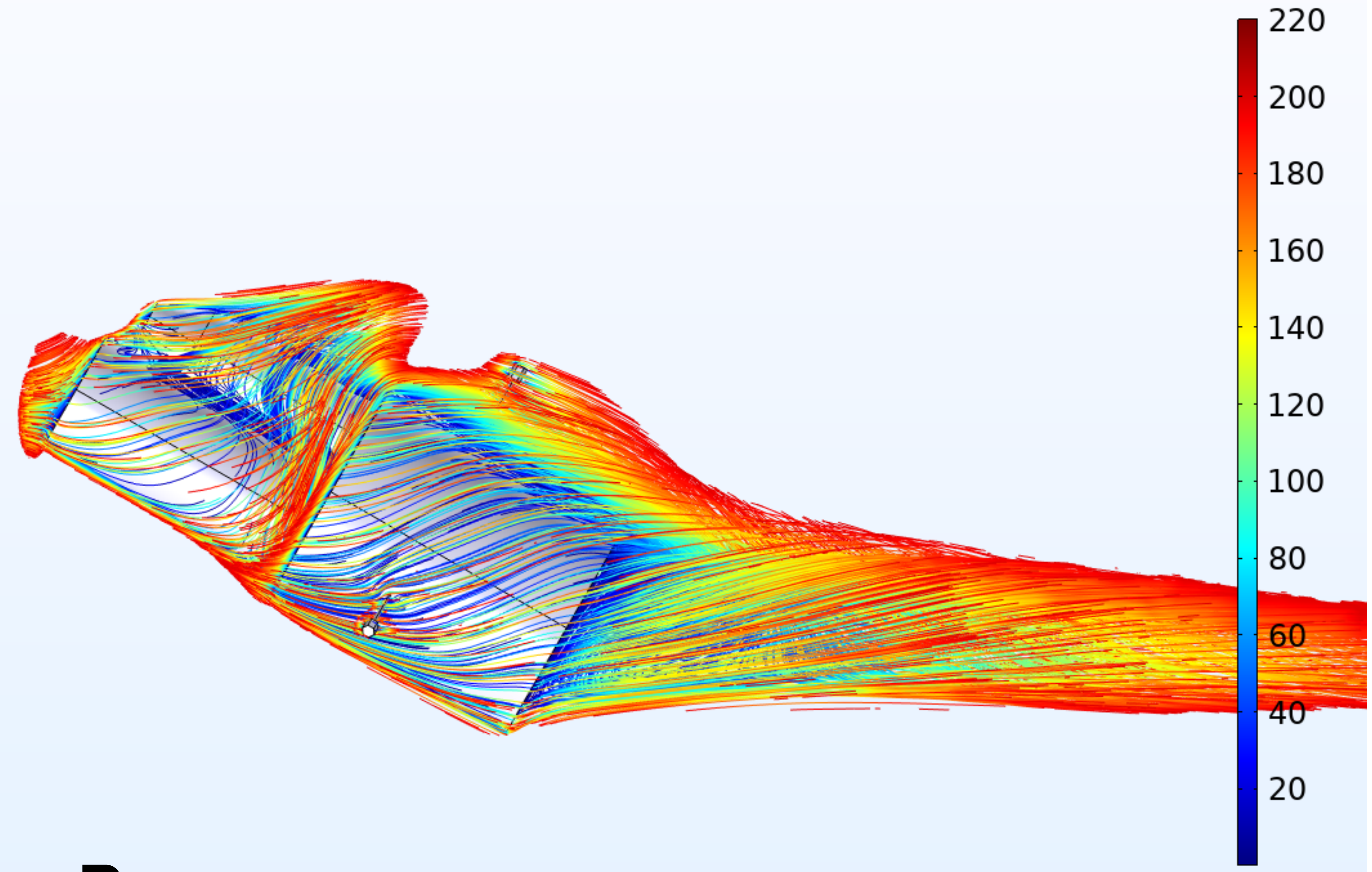

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Deutsches Zentrum für Luft- und Raumfahrt German Aerospace Center

MORABA MOBILE RAKETENBASIS

Background

The solar system is full of dust

- collisions of asteroids
- sublimation of comets (dust-laden ice) orbiting the sun
  - dust trails
  - origin of meteor showers

- long-decayed cometary trails

(Plane, Chem. Soc. Rev., 2012, 41, 6507–6518)

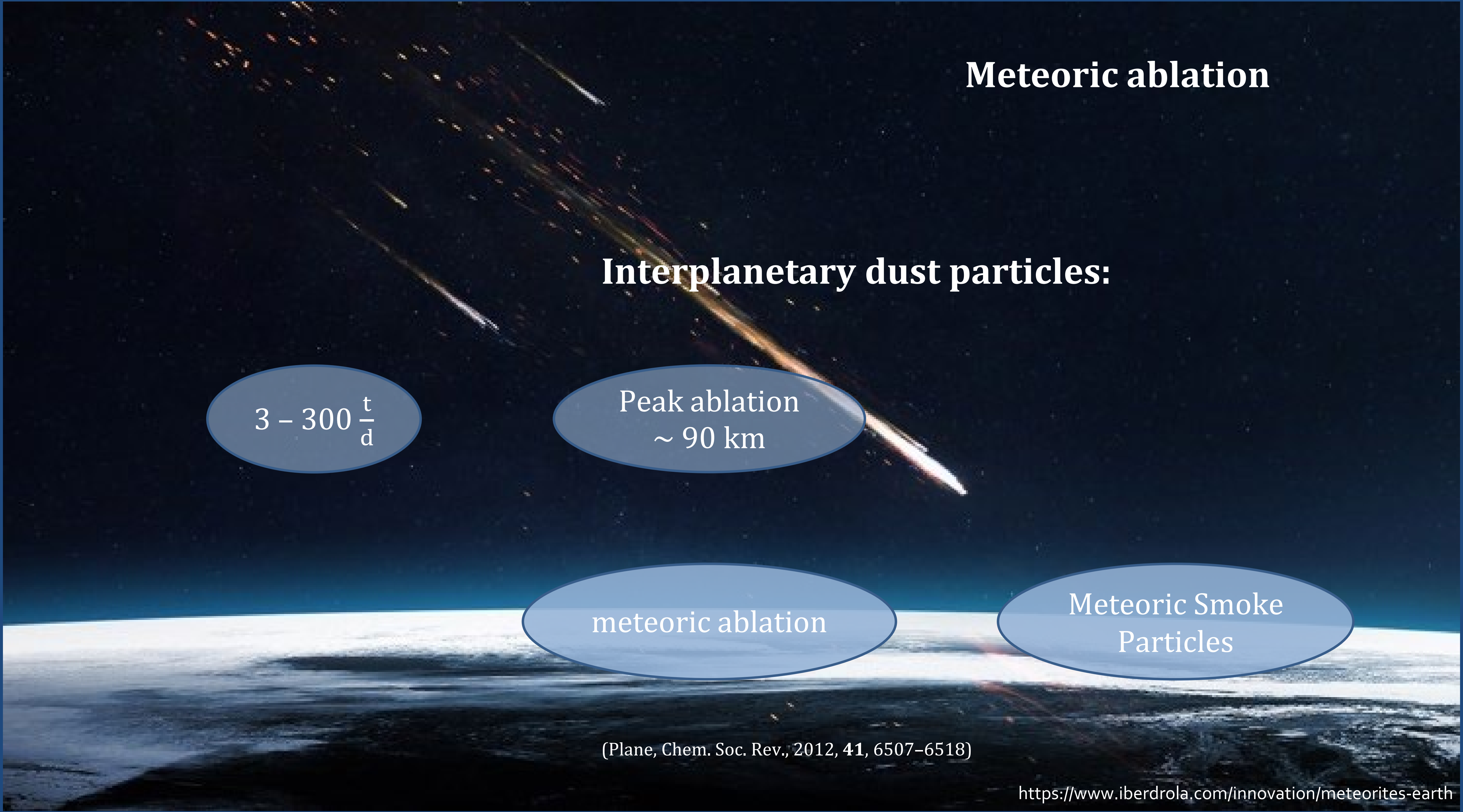
https://esahubble.org/images/potw1925a/

https://www.deccanchronicle.com/lifestyle/pets-and-environment/140516/cosmic-dust-unveils-earth-s-ancient-atmosphere.html

What is the cosmic dust input to Earth’s atmosphere?

https://www.deccanchronicle.com/lifestyle/pets-and-environment/140516/cosmic-dust-unveils-earth-s-ancient-atmosphere.html

Meteoric ablation



Interplanetary dust particles:

$3 - 300\,\frac{t}{d}$

Peak ablation
~ 90 km

meteoric ablation

Meteoric Smoke
Particles

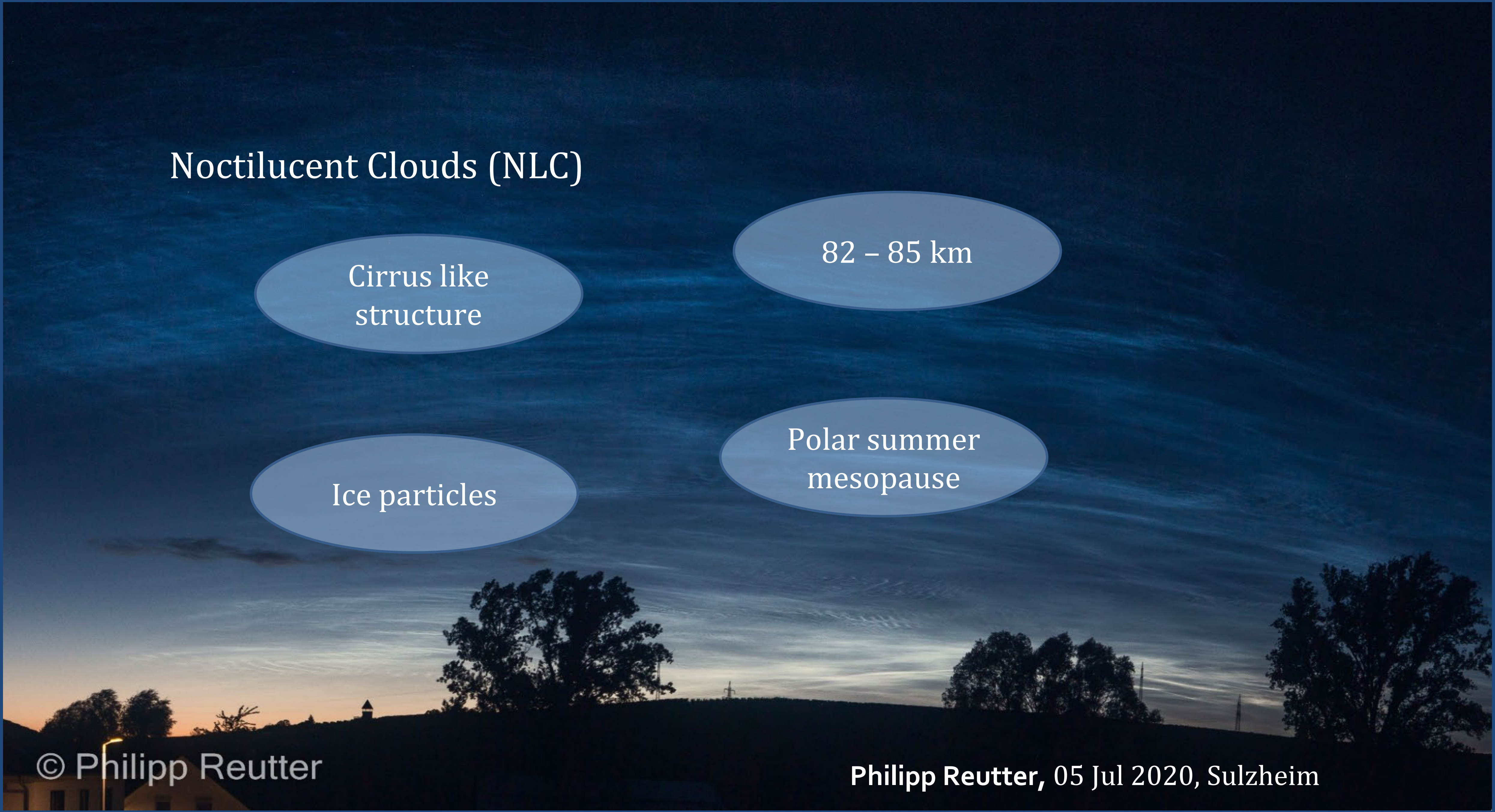
(Plane, Chem. Soc. Rev., 2012, **41**, 6507–6518)

https://www.iberdrola.com/innovation/meteorites-earth

Noctilucent Clouds (NLC)

Philipp Reutter, 05 Jul 2020, Sulzheim

Noctilucent Clouds (NLC)



Cirrus like structure

82 – 85 km

Ice particles

Polar summer mesopause

Philipp Reutter, 05 Jul 2020, Sulzheim

Do MSPs serve as ice nuclei for NLC?

Philipp Reutter, 05 Jul 2020, Sulzheim

Do MSPs serve as ice nuclei for NLC?

AIM:
Collection of mesospheric particles

© Philipp Reutter

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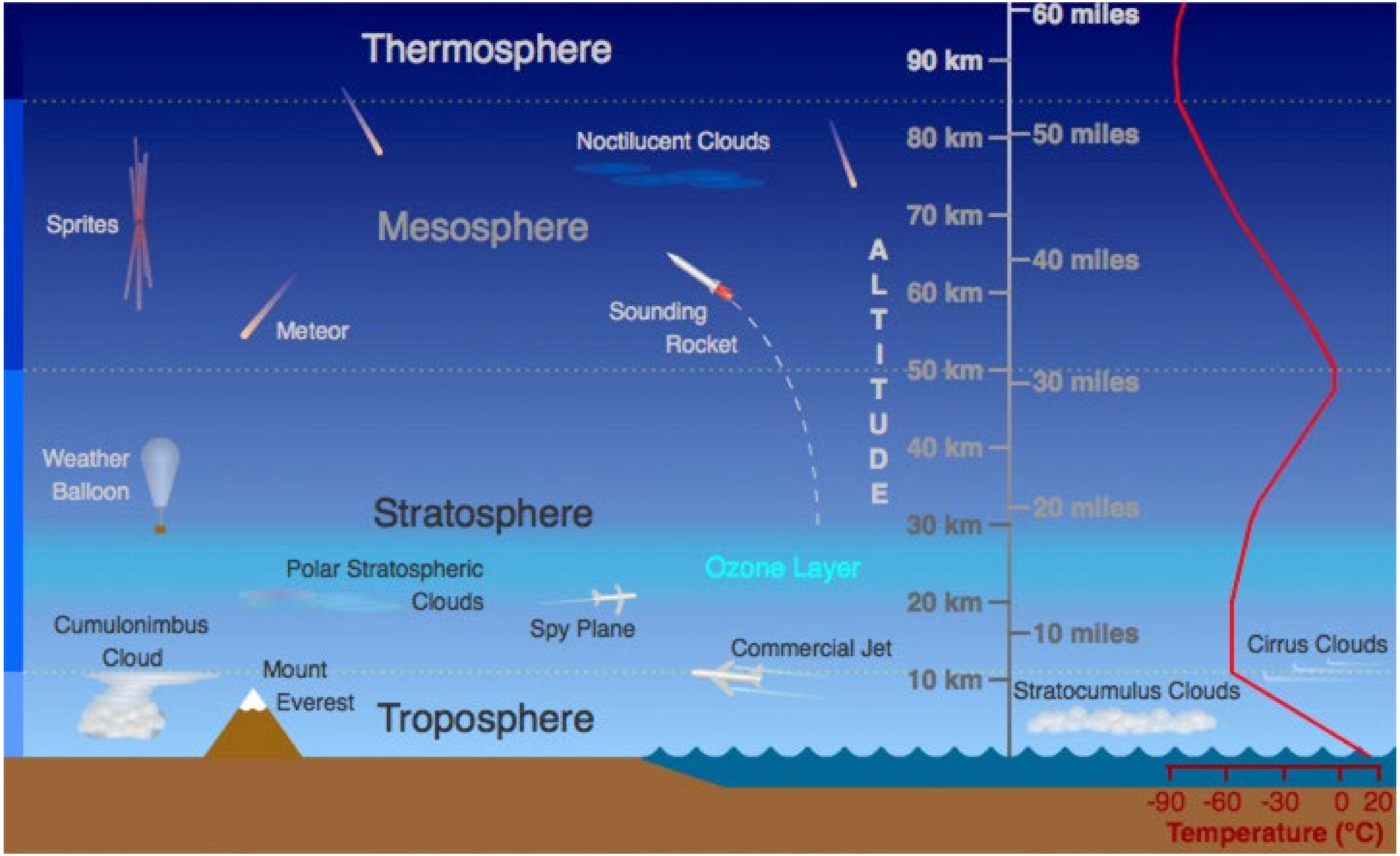
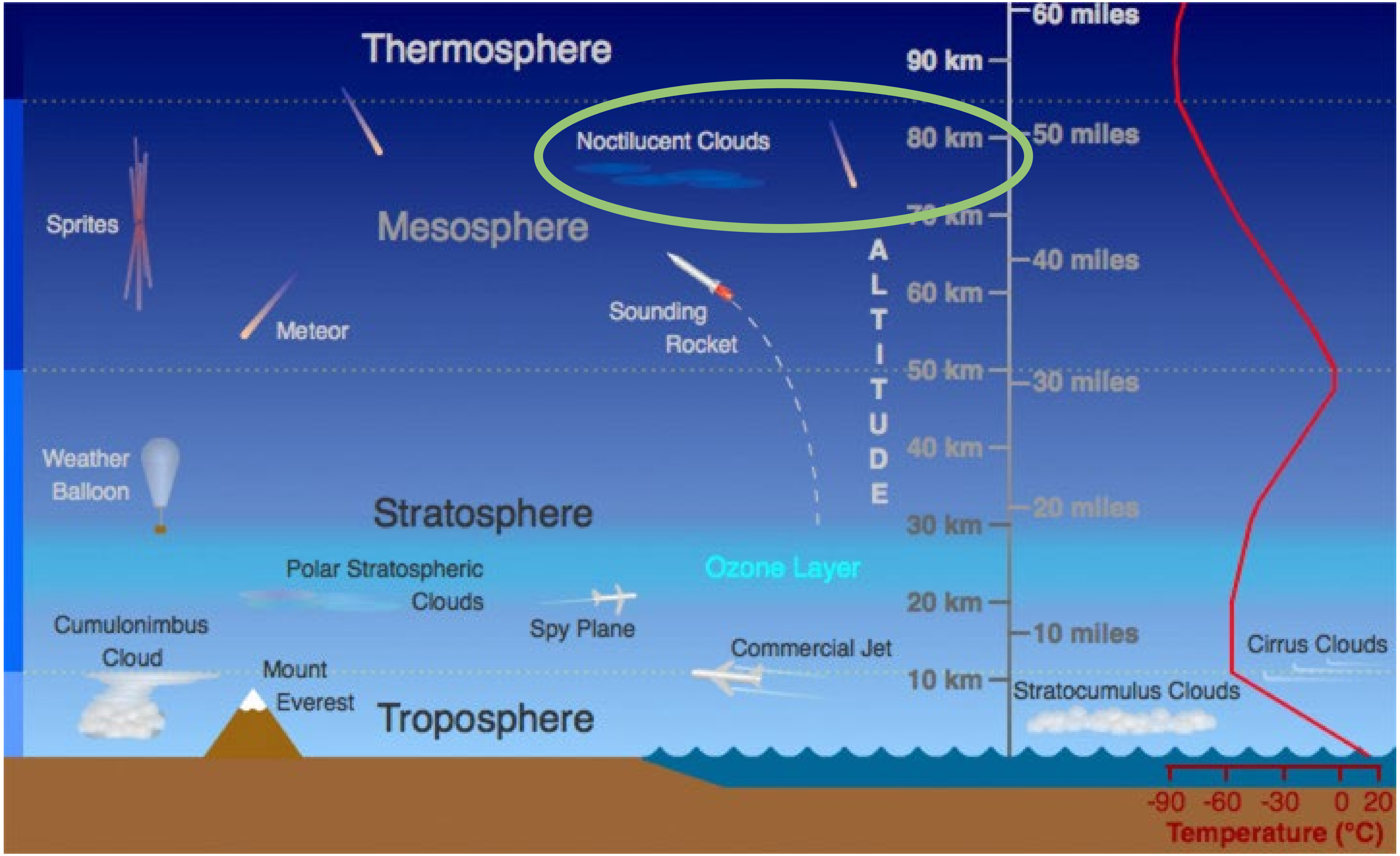


https://socratic.org/questions/with-the-help-of-a-clear-illustration-explain-the-structure-of-the-atmosphere



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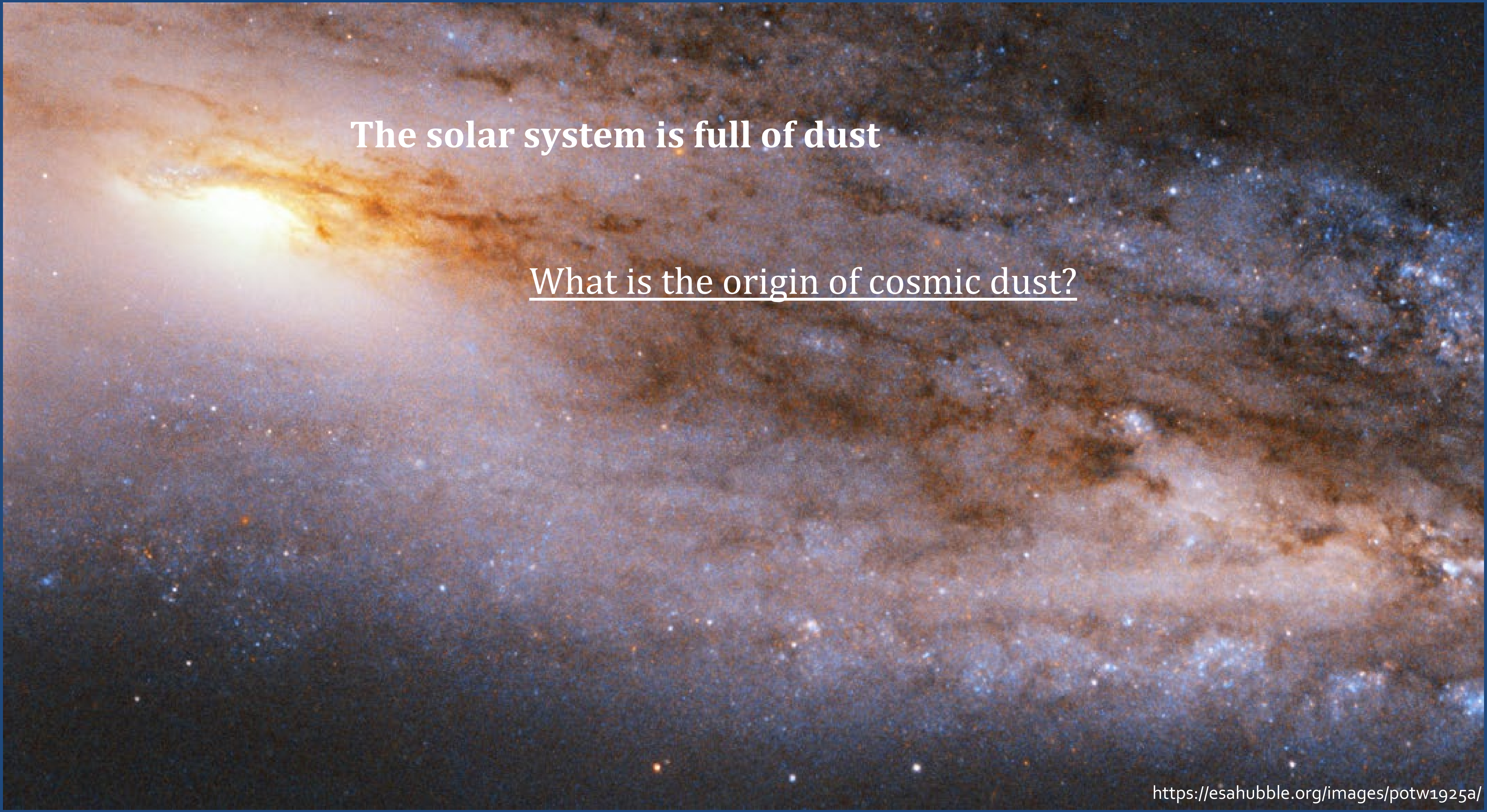
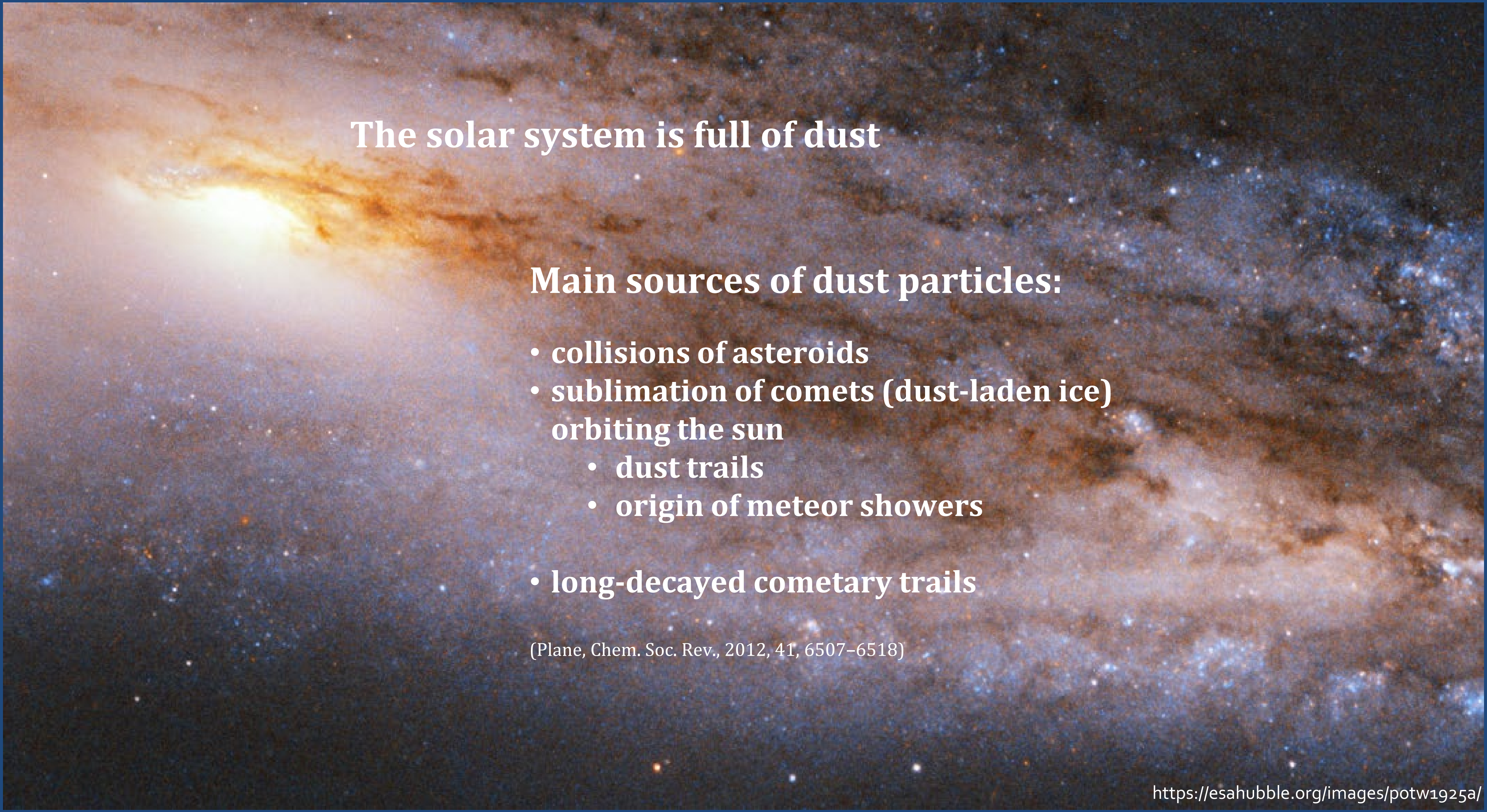


The solar system is full of dust

What is the origin of cosmic dust?

https://esahubble.org/images/potw1925a/



The solar system is full of dust

Main sources of dust particles:

- collisions of asteroids
- sublimation of comets (dust-laden ice) orbiting the sun
  - dust trails
  - origin of meteor showers

- long-decayed cometary trails

(Plane, Chem. Soc. Rev., 2012, 41, 6507–6518)

https://esahubble.org/images/potw1925a/

Meteoric ablation

Na

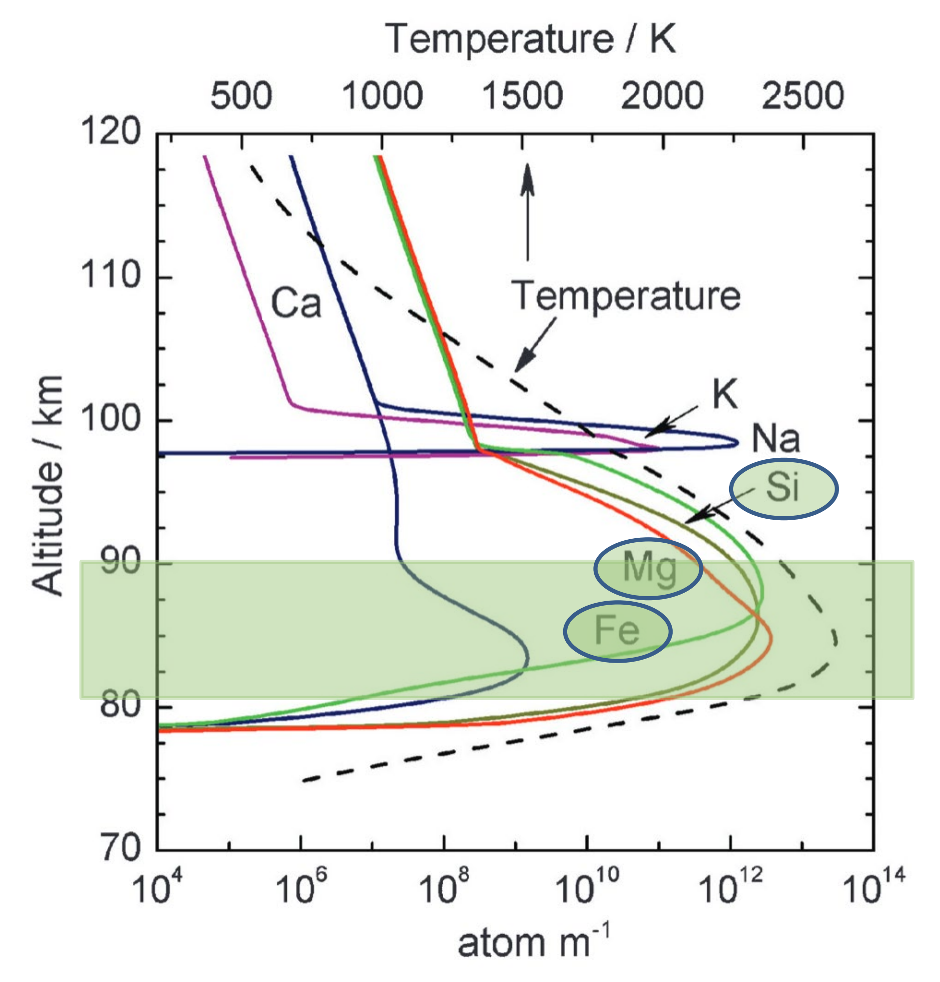
K

Fe

Mg

Si

Ca



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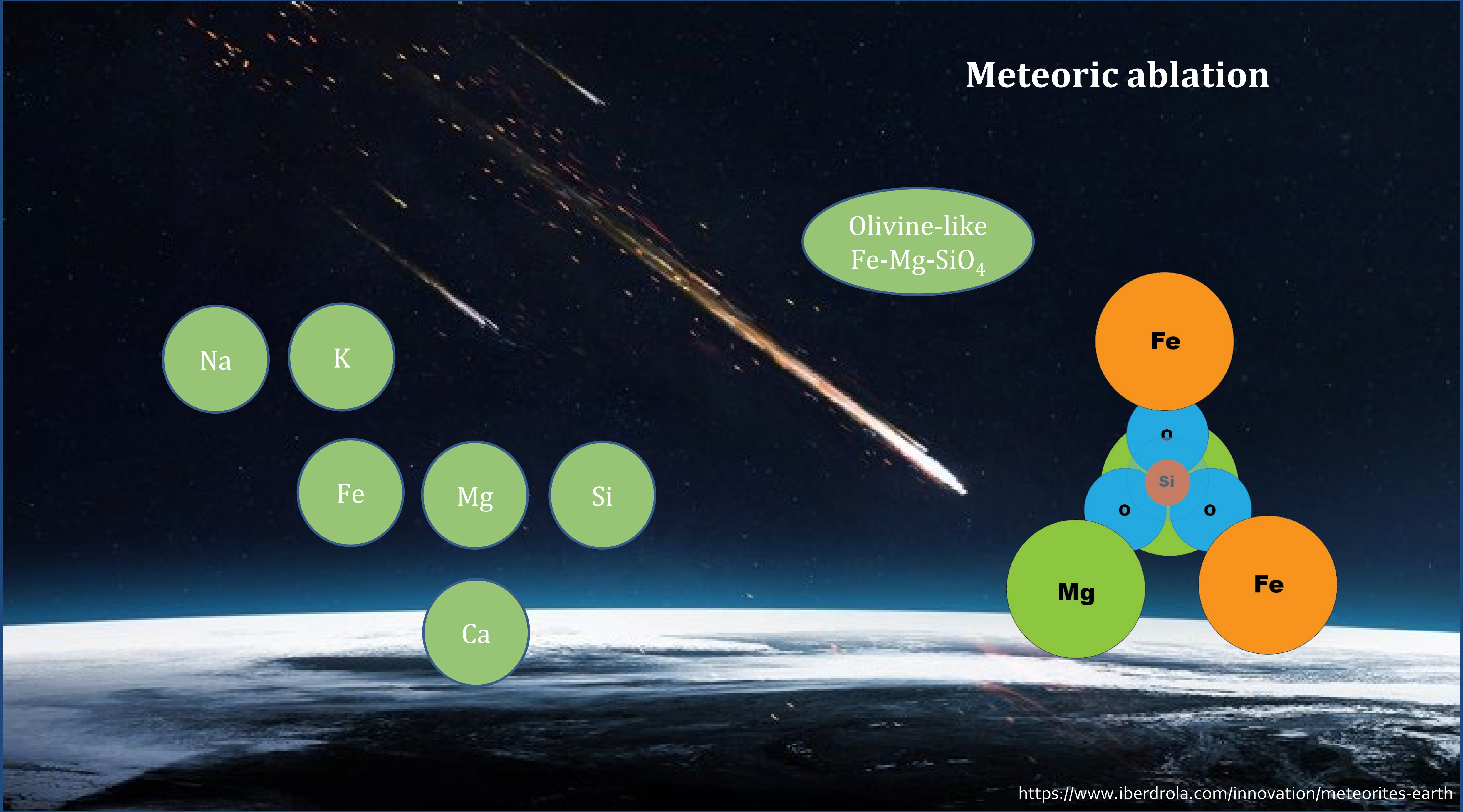
Meteoric ablation



Na

K

Fe

Mg

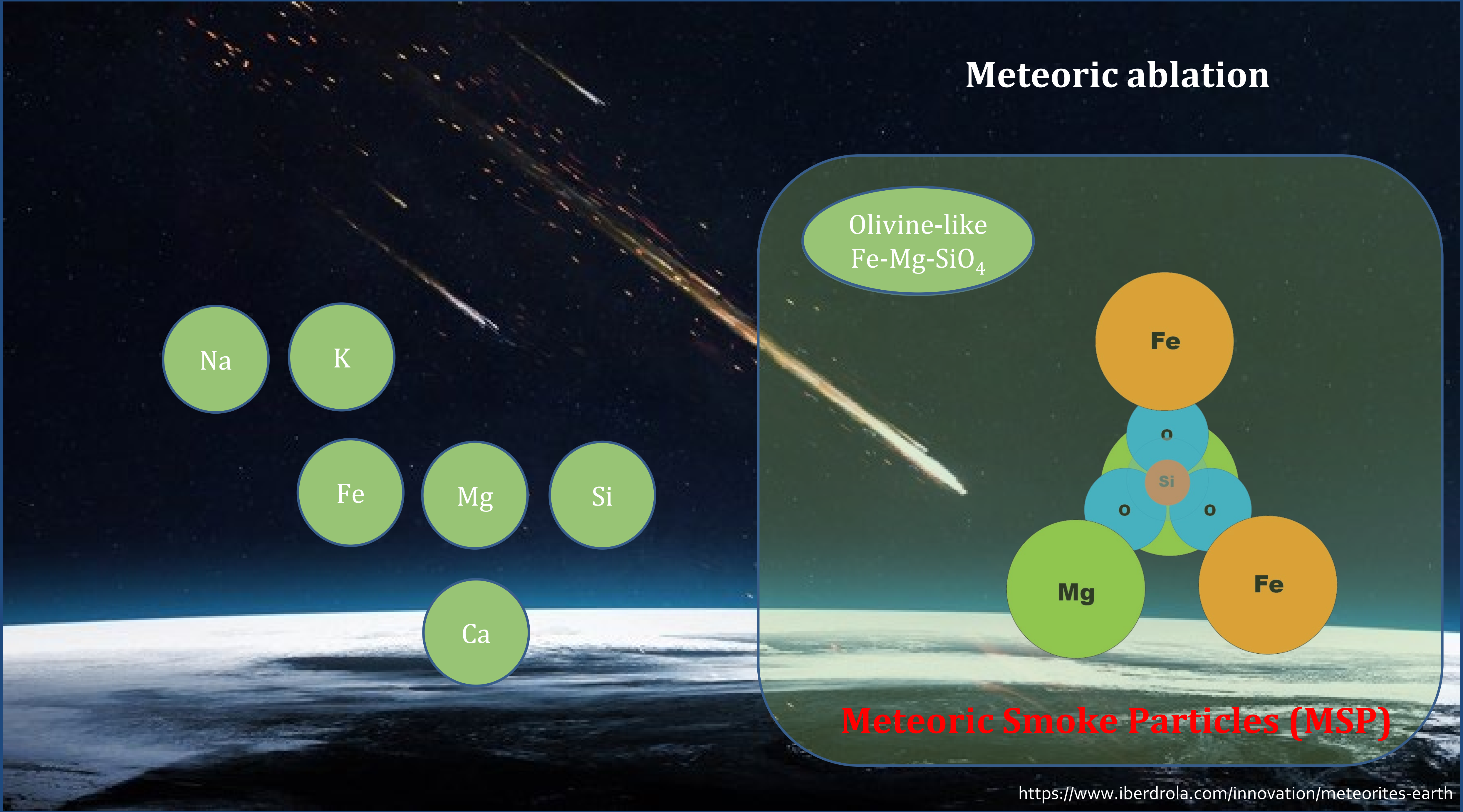
Si

Ca

Olivine-like
Fe-Mg-SiO_4

https://www.iberdrola.com/innovation/meteorites-earth

Meteoric ablation



Na K

Fe Mg Si

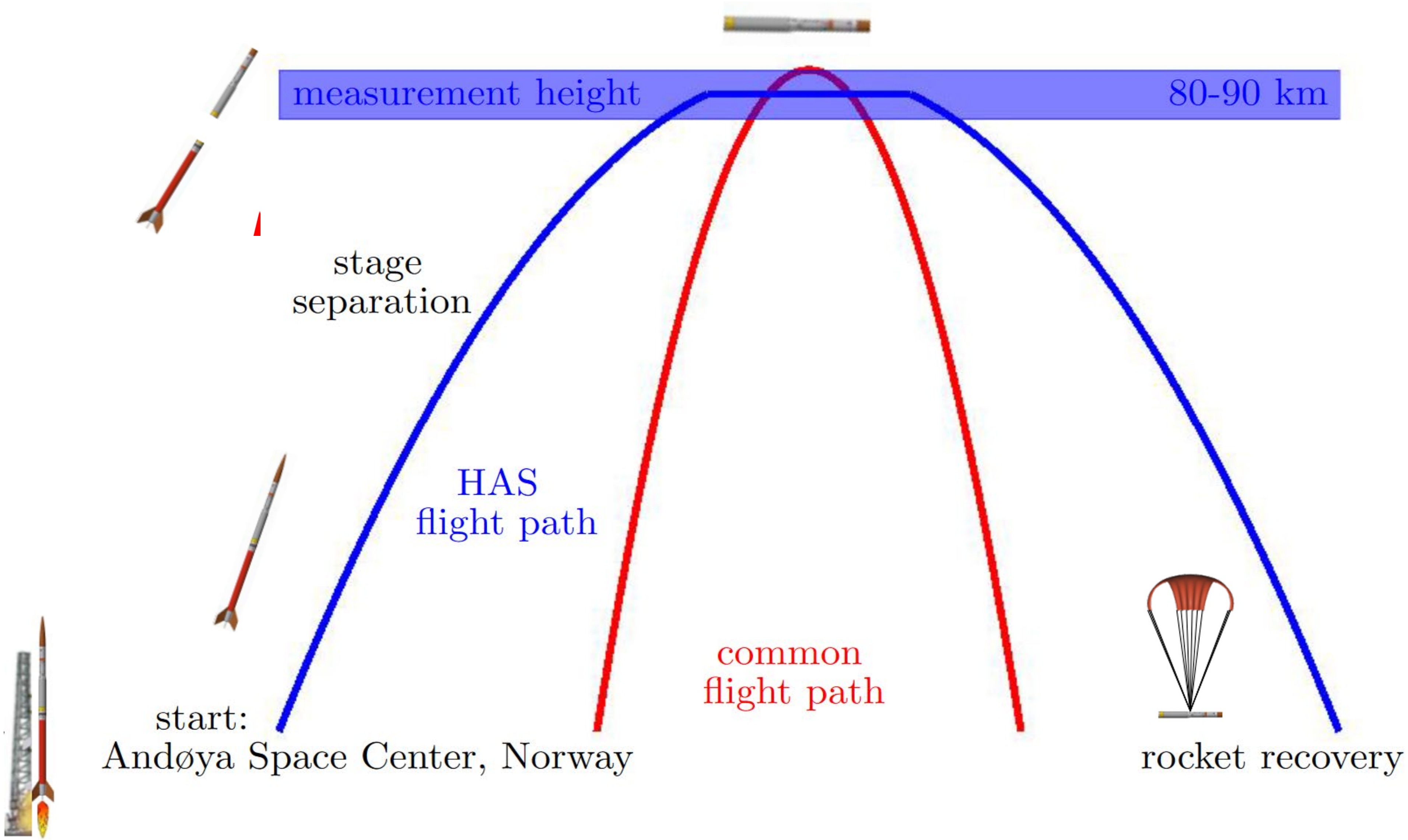
Ca

Olivine-like
Fe-Mg-SiO$_4$

Meteoric Smoke Particles (MSP)

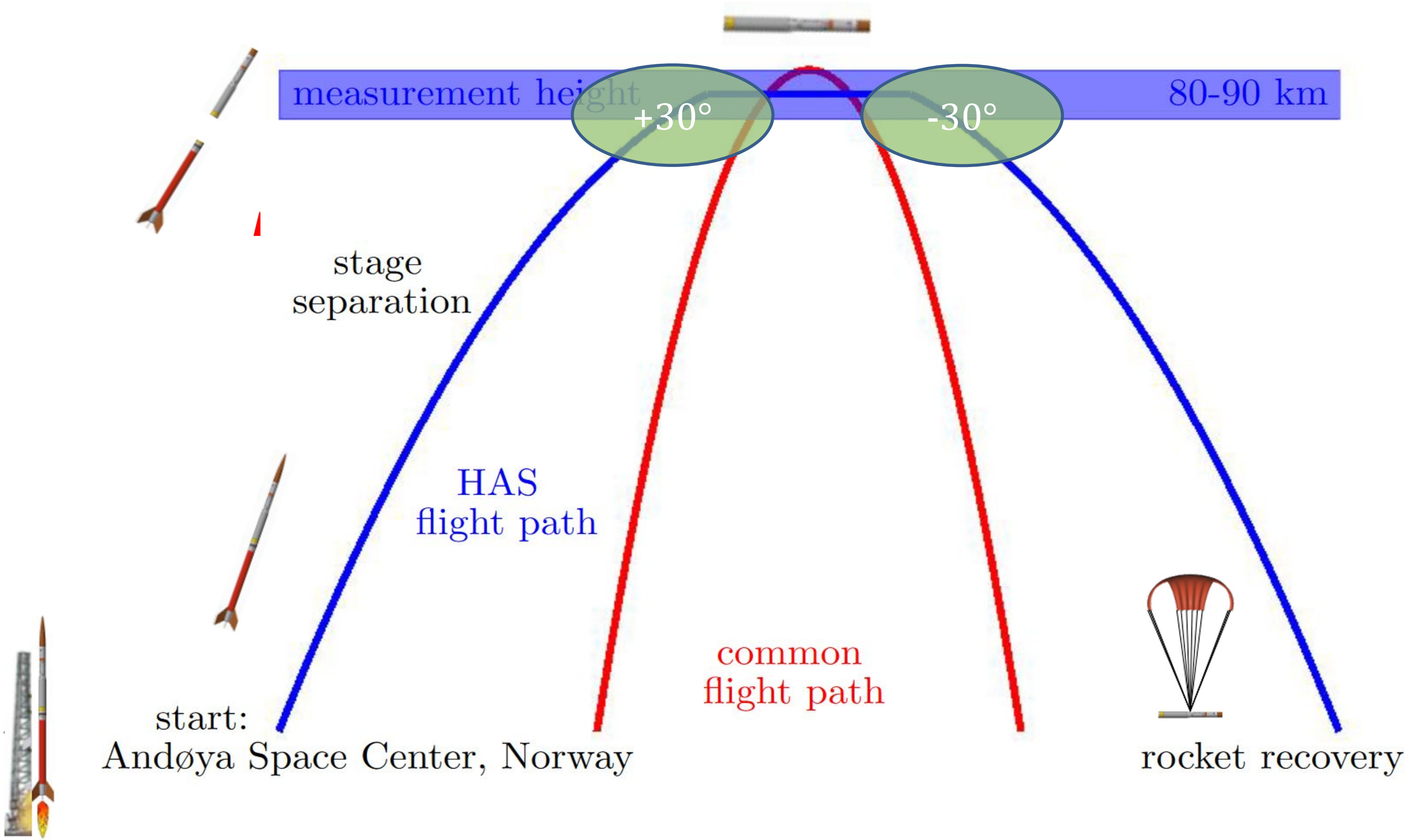
https://www.iberdrola.com/innovation/meteorites-earth

Flight path of the rocket



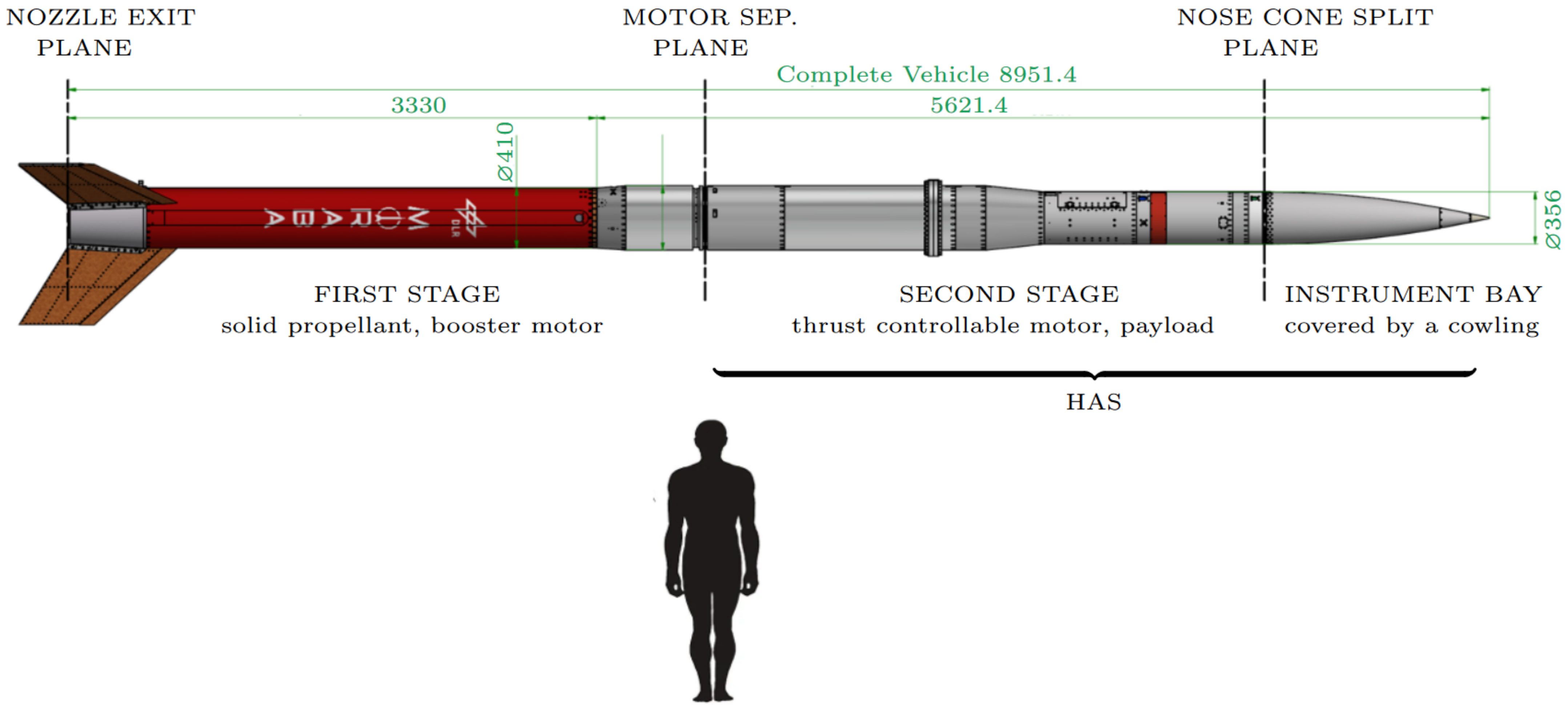
adopted from Naumann et al.; https://elib.dlr.de/140301/1/485_NAUMANN-DRESCHER.pdf

Flight path of the sounding rocket



adopted from Naumann et al.; https://elib.dlr.de/140301/1/485_NAUMANN-DRESCHER.pdf

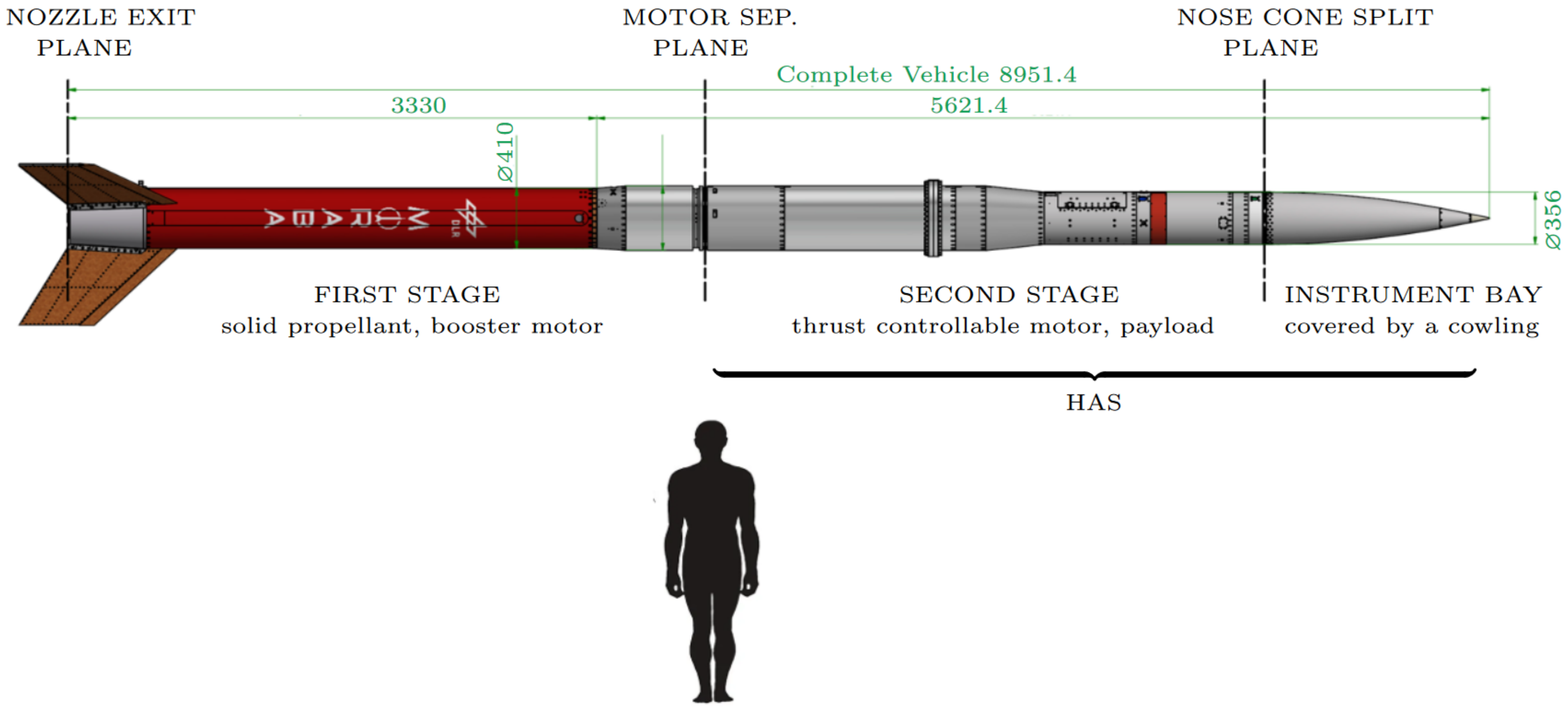
Sounding rocket



adopted from Naumann, K., et al. "Design of a hovering sounding rocket stage for measurements in the high atmosphere." (2020)

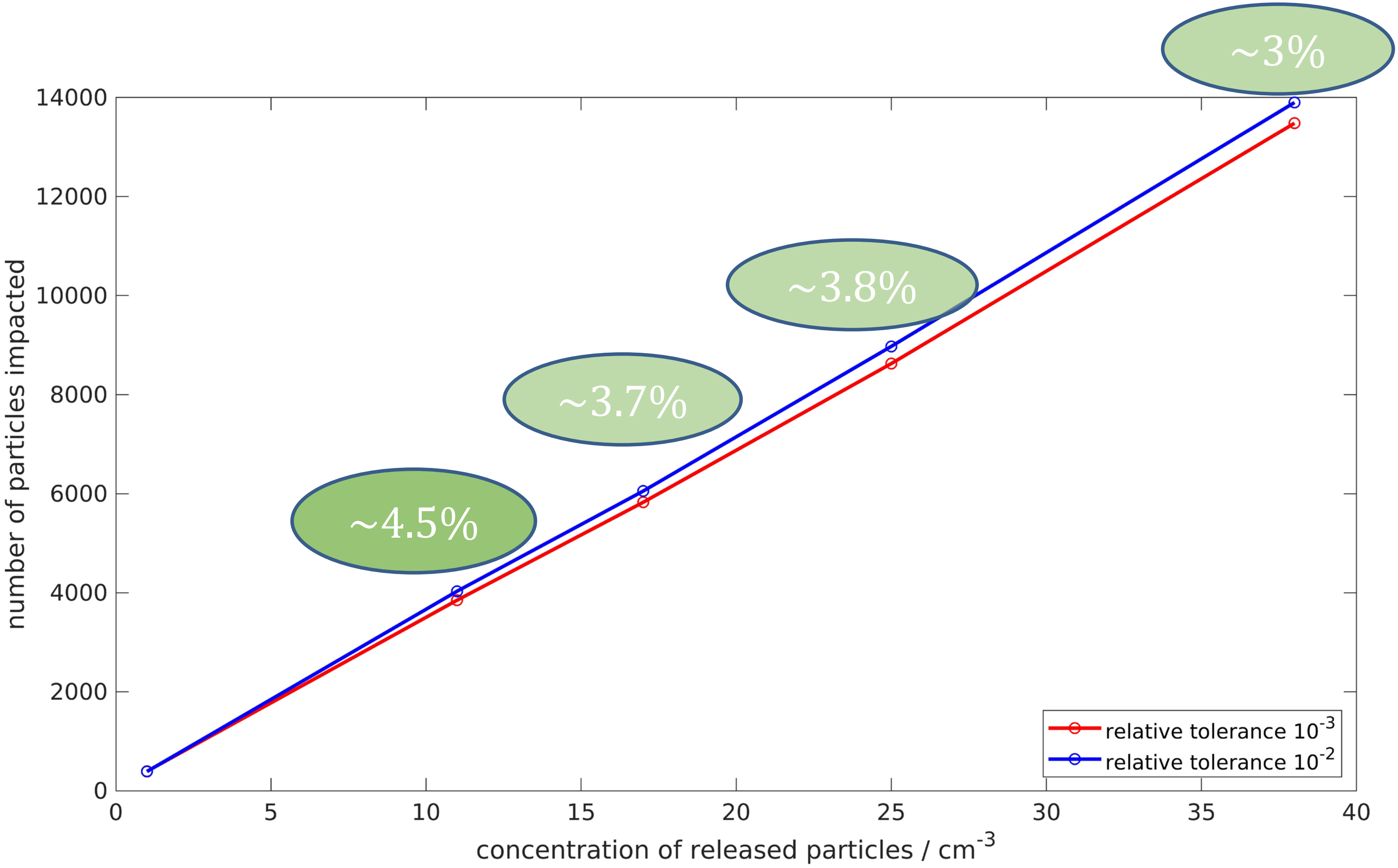
thrust controllable experiment carrier

DLR-Moraba



adopted from Naumann, K., et al. "Design of a hovering sounding rocket stage for measurements in the high atmosphere." (2020)

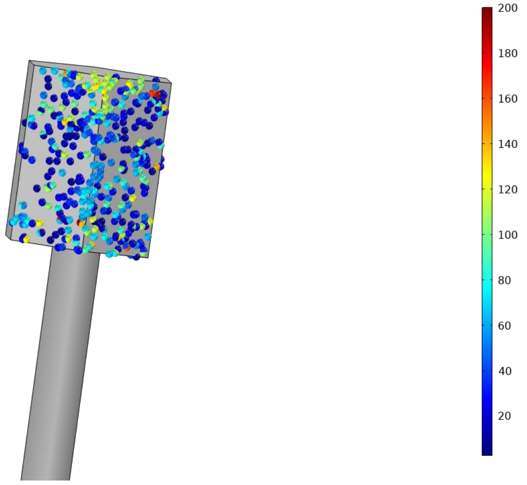
Particle simulations



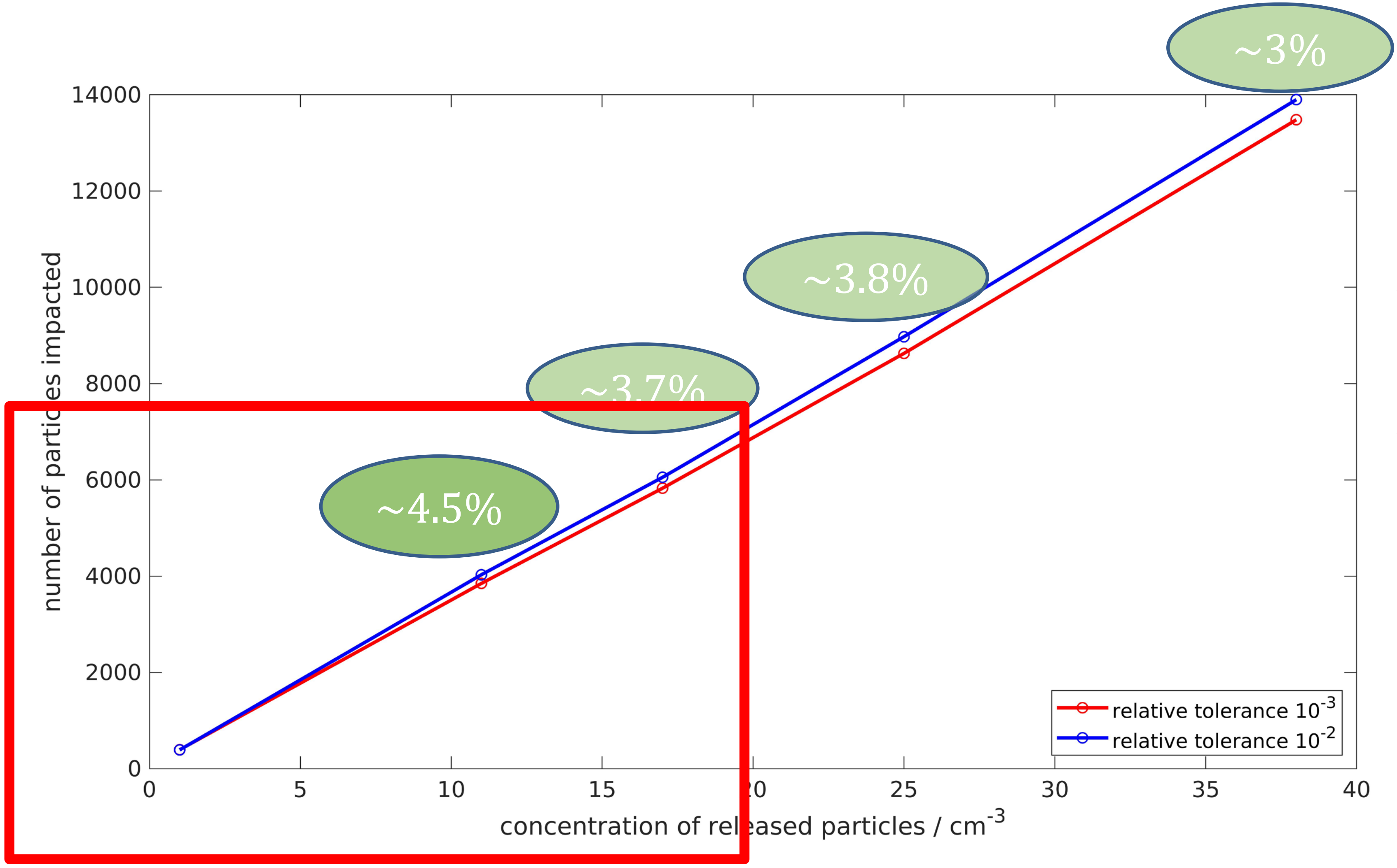
Relative tolerance Tol

$Tol = 10^{-3}$ with $\Delta t \sim 3 \cdot 10^{-6}$ s
Simulation time: ~90 h

$Tol = 10^{-2}$ with $\Delta t \sim 2 \cdot 10^{-5}$ s
Simulation time: ~30 h



Particle simulations



Relative tolerance Tol

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Simulation time: ~90 h

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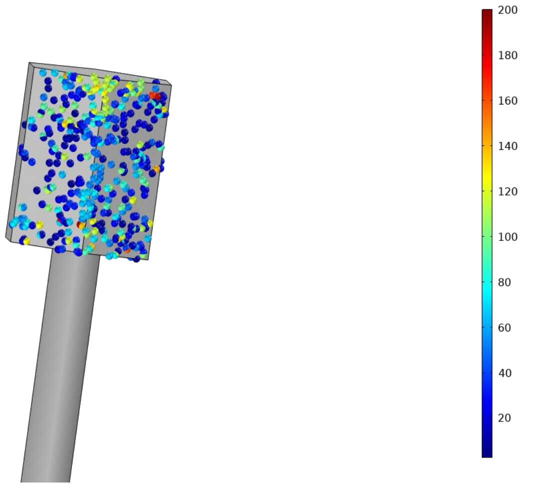
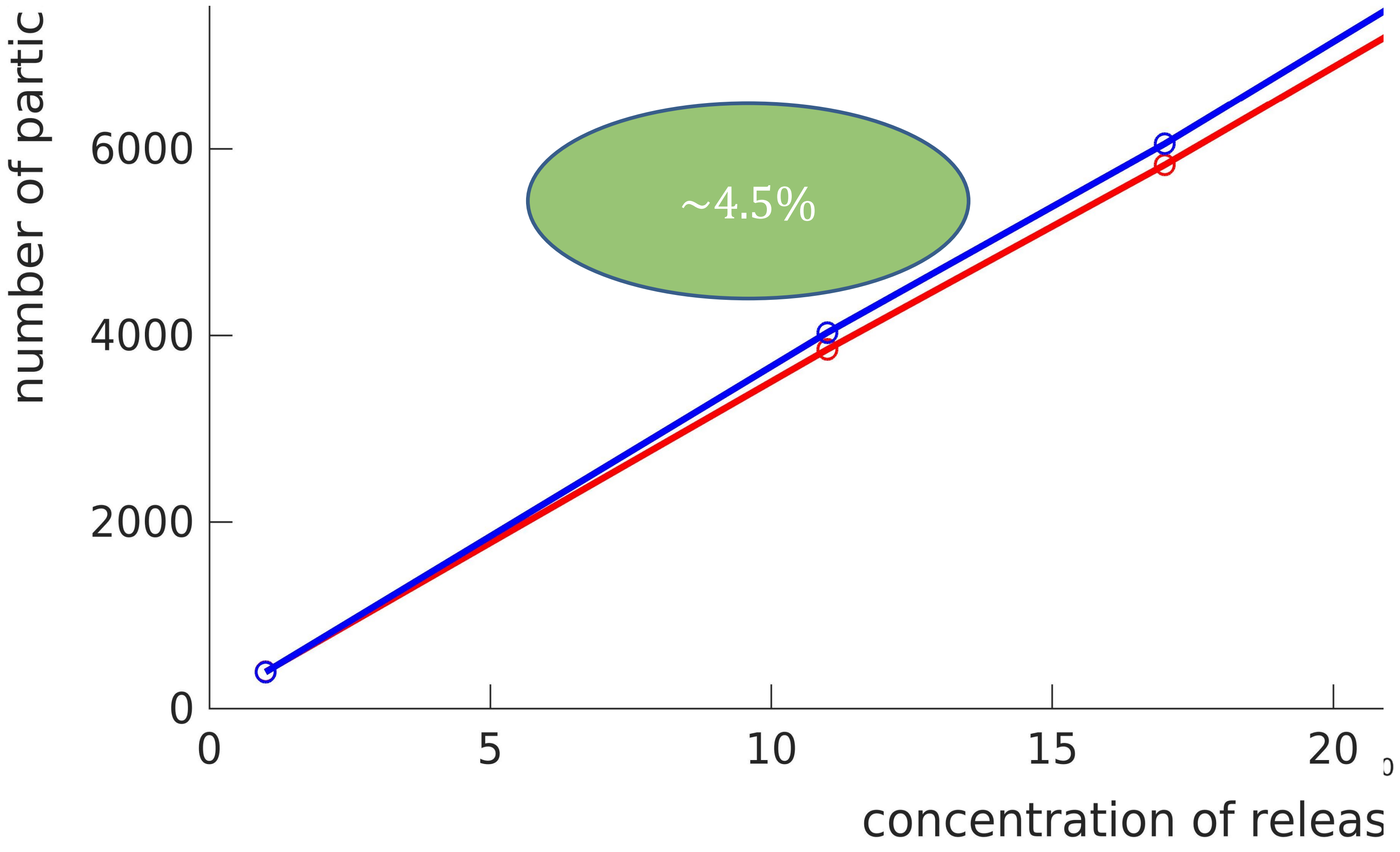
Simulation time: ~30 h



Particle simulations

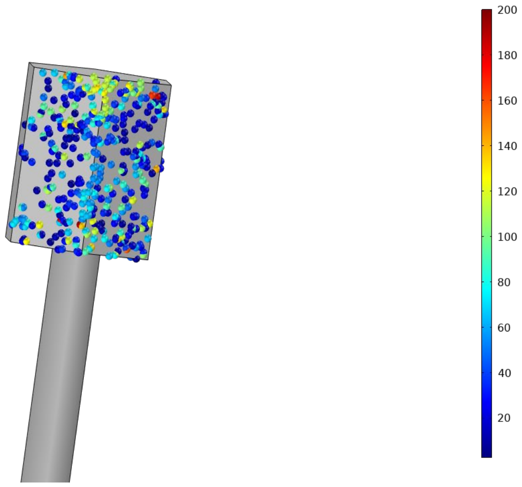


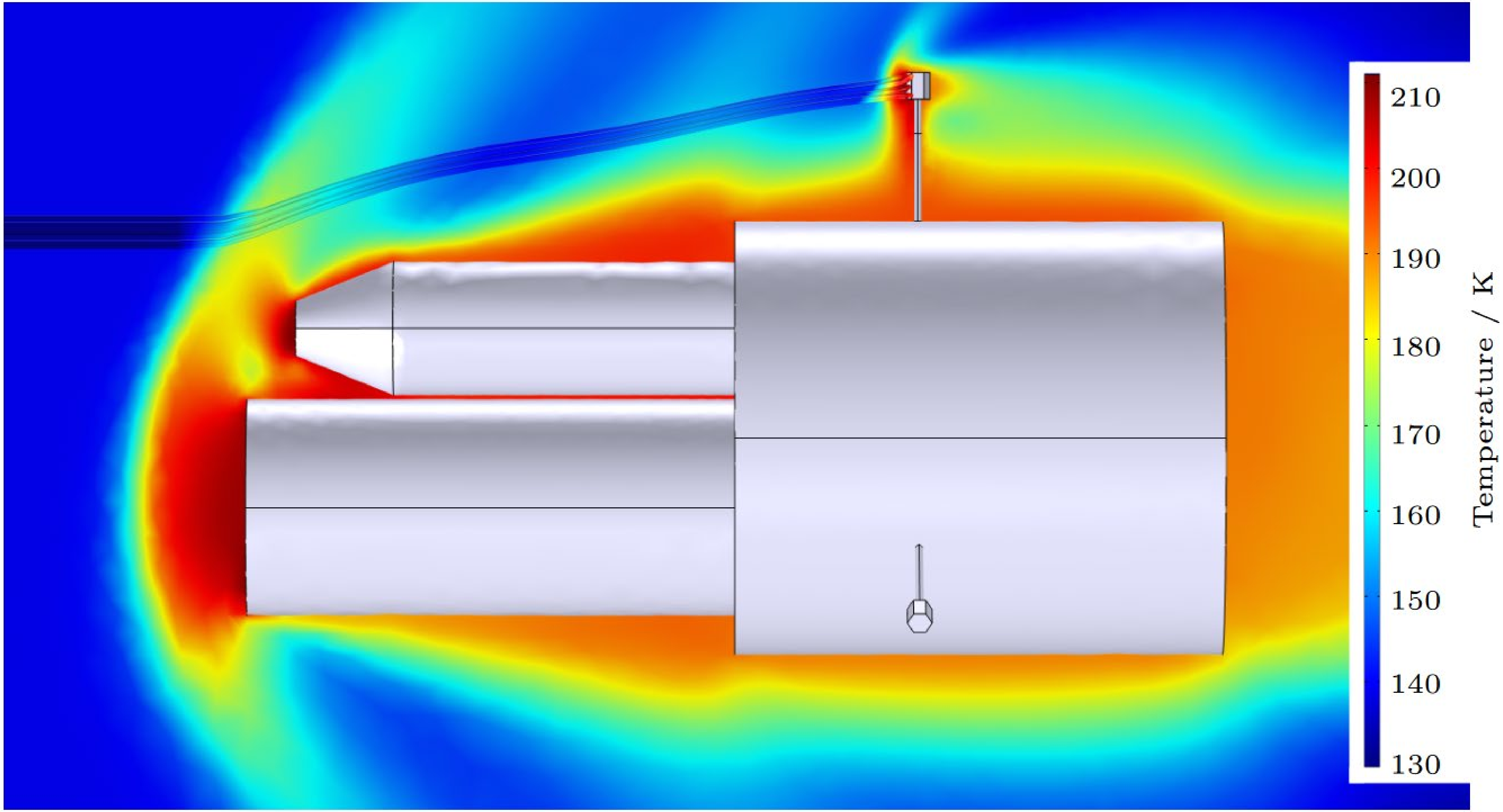
~4.5%

Relative tolerance Tol

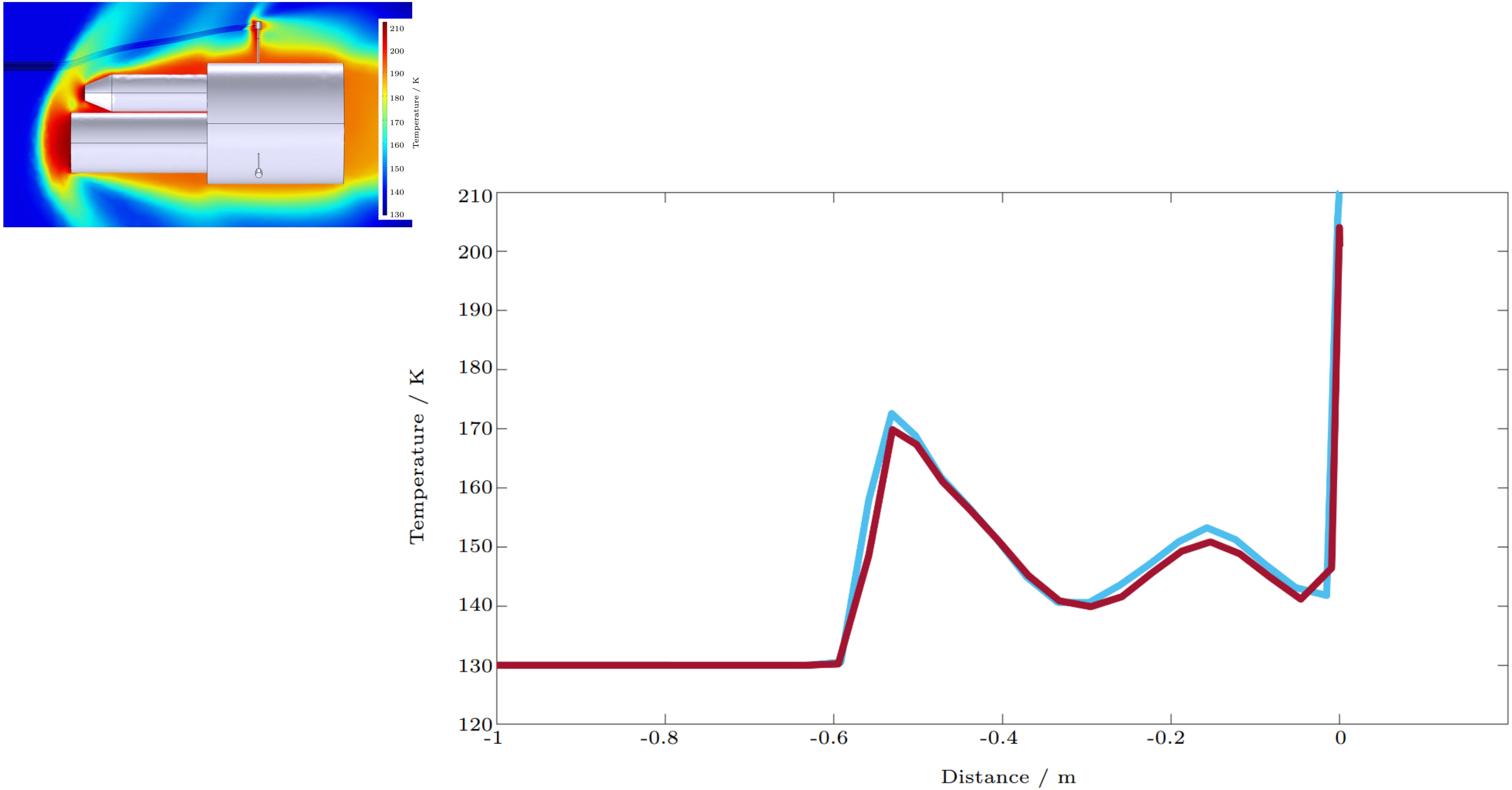
$Tol = 10^{-3}$ with $\Delta t \sim 3 \cdot 10^{-6}$ s
Simulation time: ~90 h

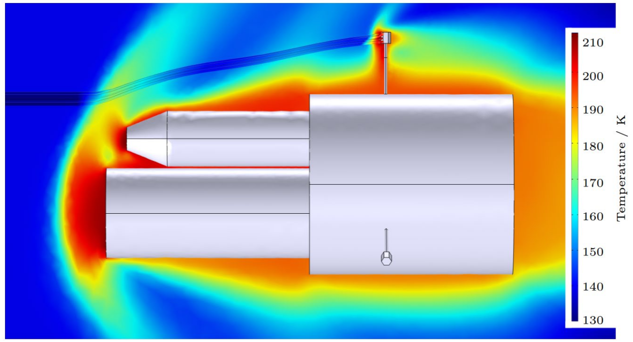
$Tol = 10^{-2}$ with $\Delta t \sim 2 \cdot 10^{-5}$ s
Simulation time: ~30 h



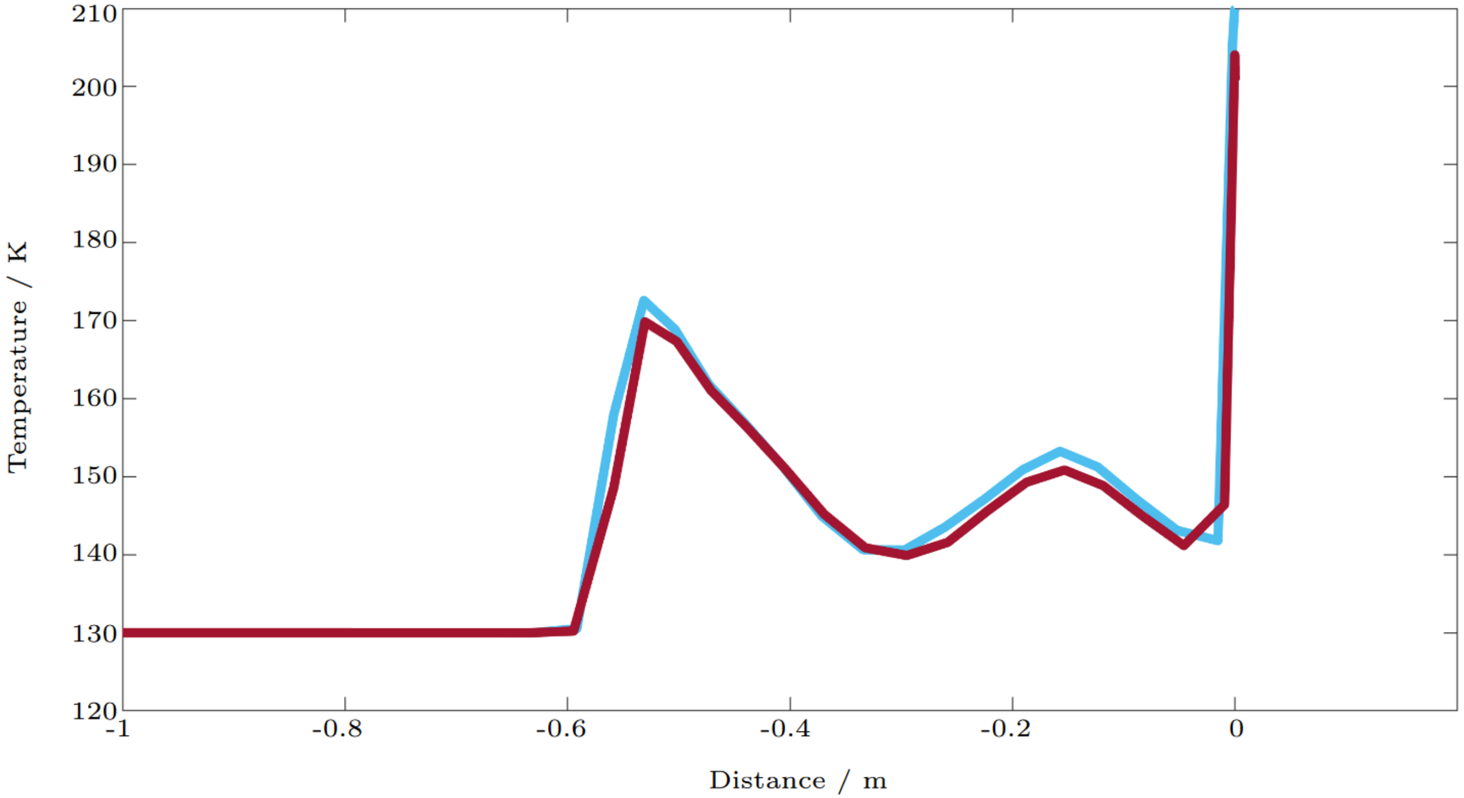


Temperature field in K depicted on a cut plane and particle trajectories


210
200
190
180
170
160
150
140
130
Temperature / K
210
200
190
180
170
160
150
140
130
120
Temperature / K
-1
-0.8
-0.6
-0.4
-0.2
0
Distance / m



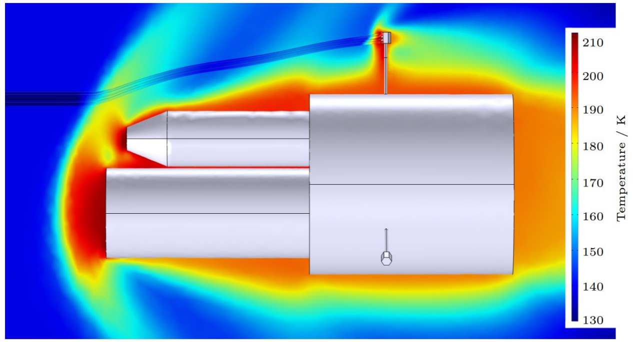
$$\frac{dm_{ice}}{dt} = \frac{4\pi}{R_v}\, \alpha_{\mathrm{d}}\, D_v\, r \left(\frac{p}{T_{air}} - \frac{p}{T_{ice}}\right)$$



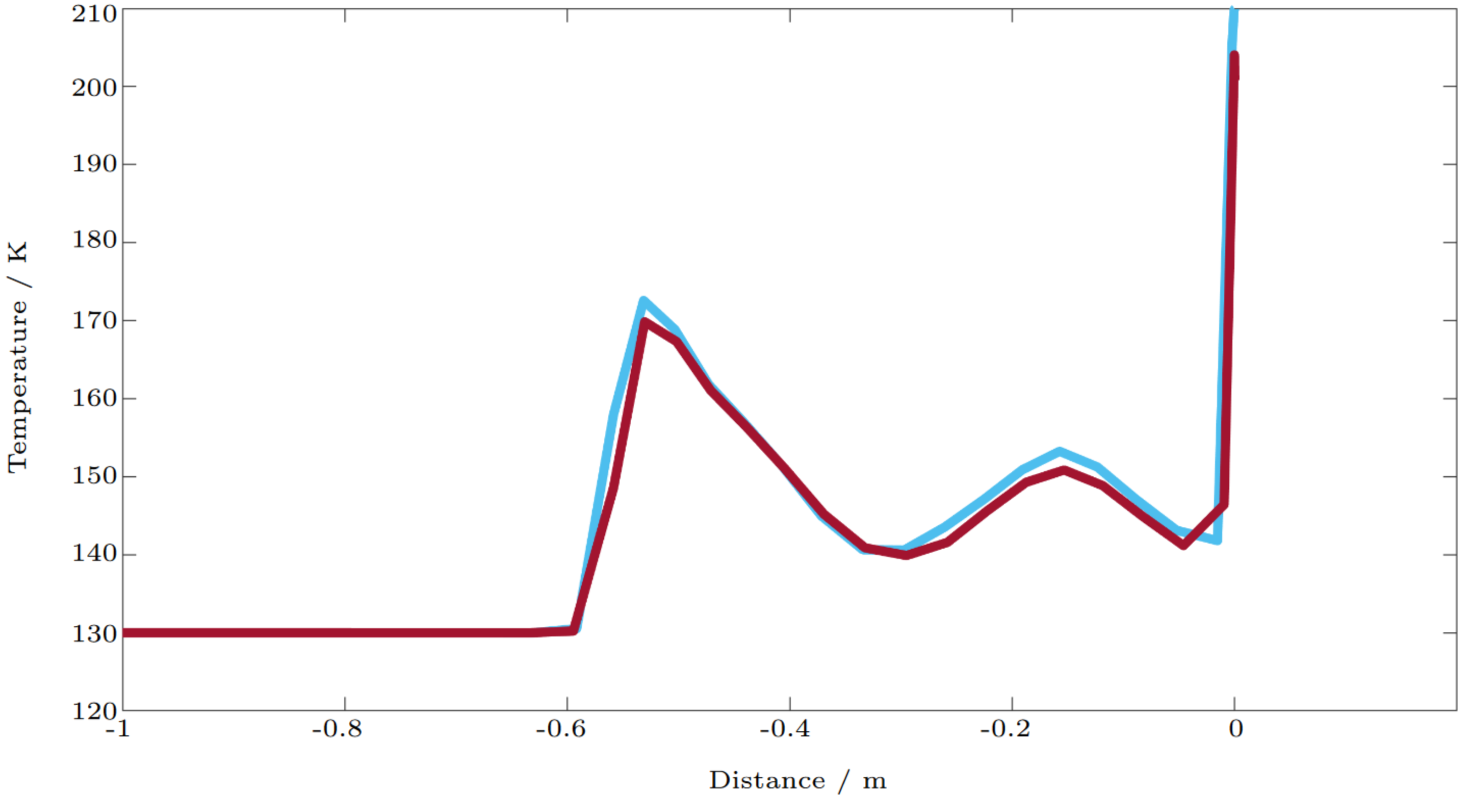
D_v: diffusion coefficient of water vapor in air
p: pressure
r: particle radius

T_{air}: ambient temperature
T_{ice}: surface temperature
α_{d}: deposition coefficient

R_v: water vapor gas constant
p: pressure
r: particle radius



$$\frac{dm_{ice}}{dt} = \frac{4\pi}{R_v} \alpha_d D_v \, r \left(\frac{p}{T_{air}} - \frac{p}{T_{ice}} \right)$$



D_v: diffusion coefficient of water vapor in air
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R_v: water vapor gas constant
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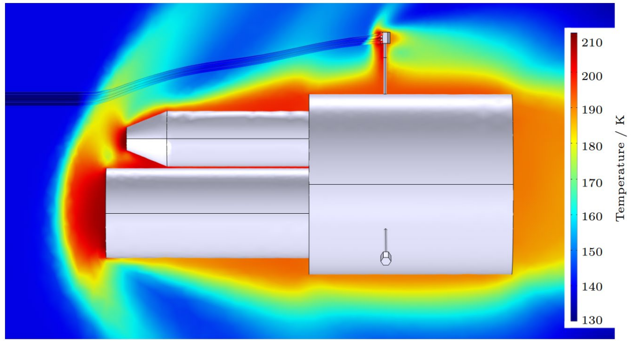

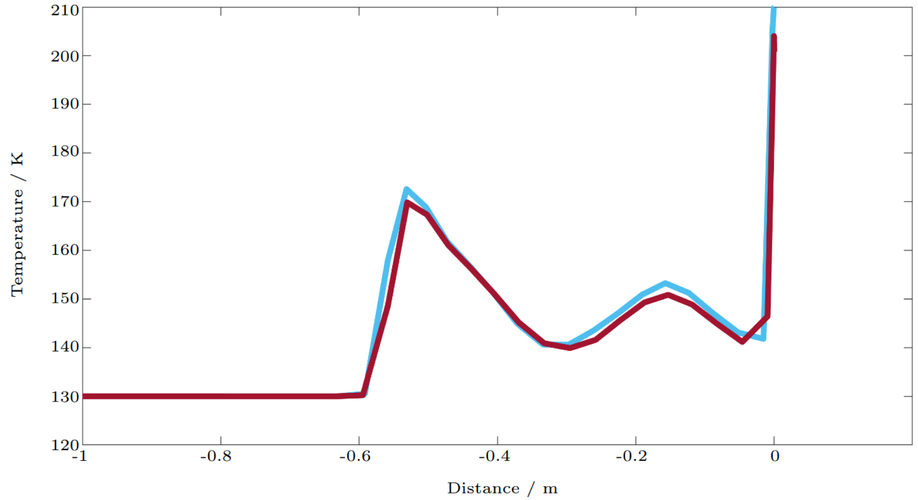

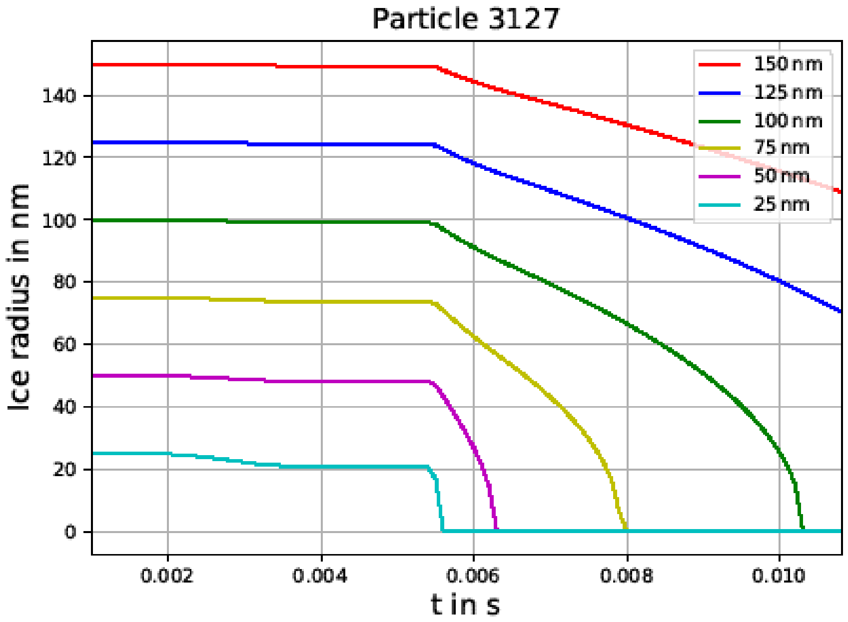

$$\frac{dm_{ice}}{dt} = \frac{4\pi}{R_v} \alpha_d D_v \, r \left(\frac{p}{T_{air}} - \frac{p}{T_{ice}} \right)$$

D_v: diffusion coefficient of water vapor in air
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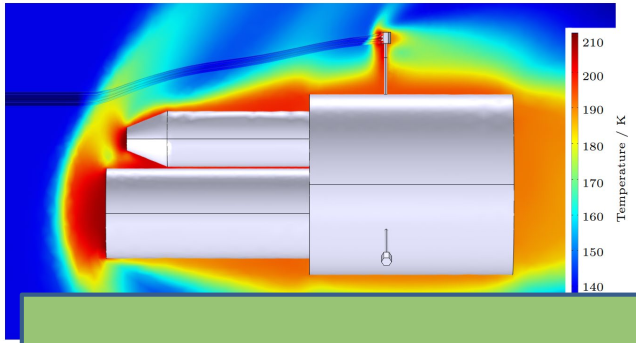
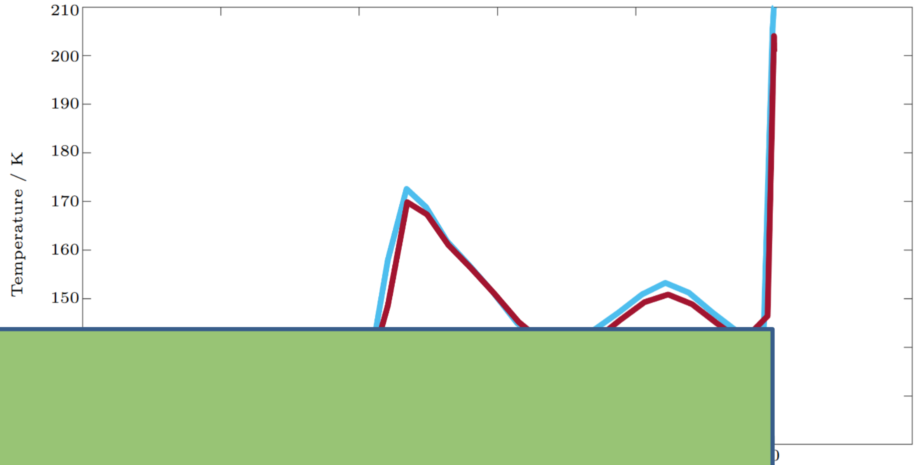
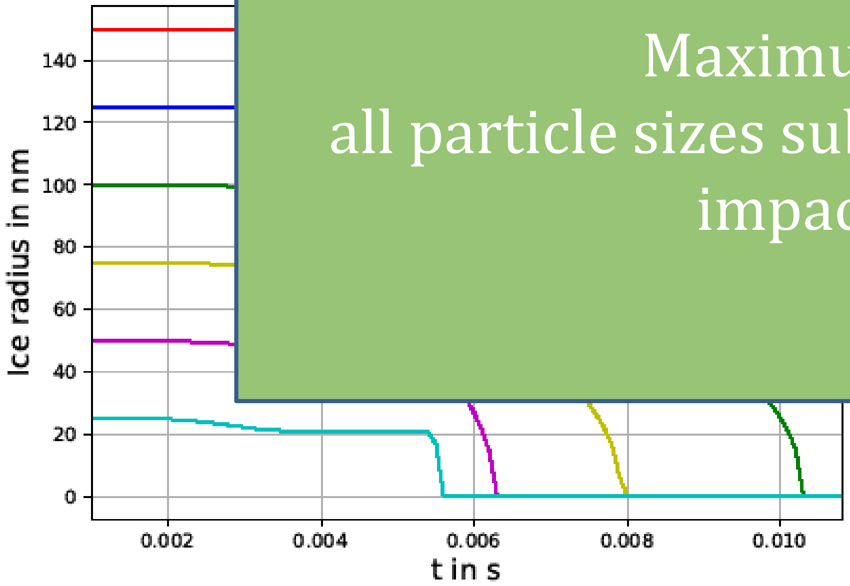






Maximum assumption:
all particle sizes sublimate on their way to the impaction surfaces

$$\left(\frac{p}{T_{air}} - \frac{p}{T_{ice}}\right)$$

D_v: diffusion coefficient of water vapor in air
p: pressure
r: particle radius

T_{air}: ambient temperature
T_{ice}: surface temperature
α_d: deposition coefficient

R_v: water vapor gas constant
p: pressure
r: particle radius

Particle data

NLC particle

$\rho_p = 1\ \dfrac{\mathrm{g}}{\mathrm{cm}^3}$

$r_p \leq 10\ \mu\mathrm{m}$

C=1-10 cm^{-3}

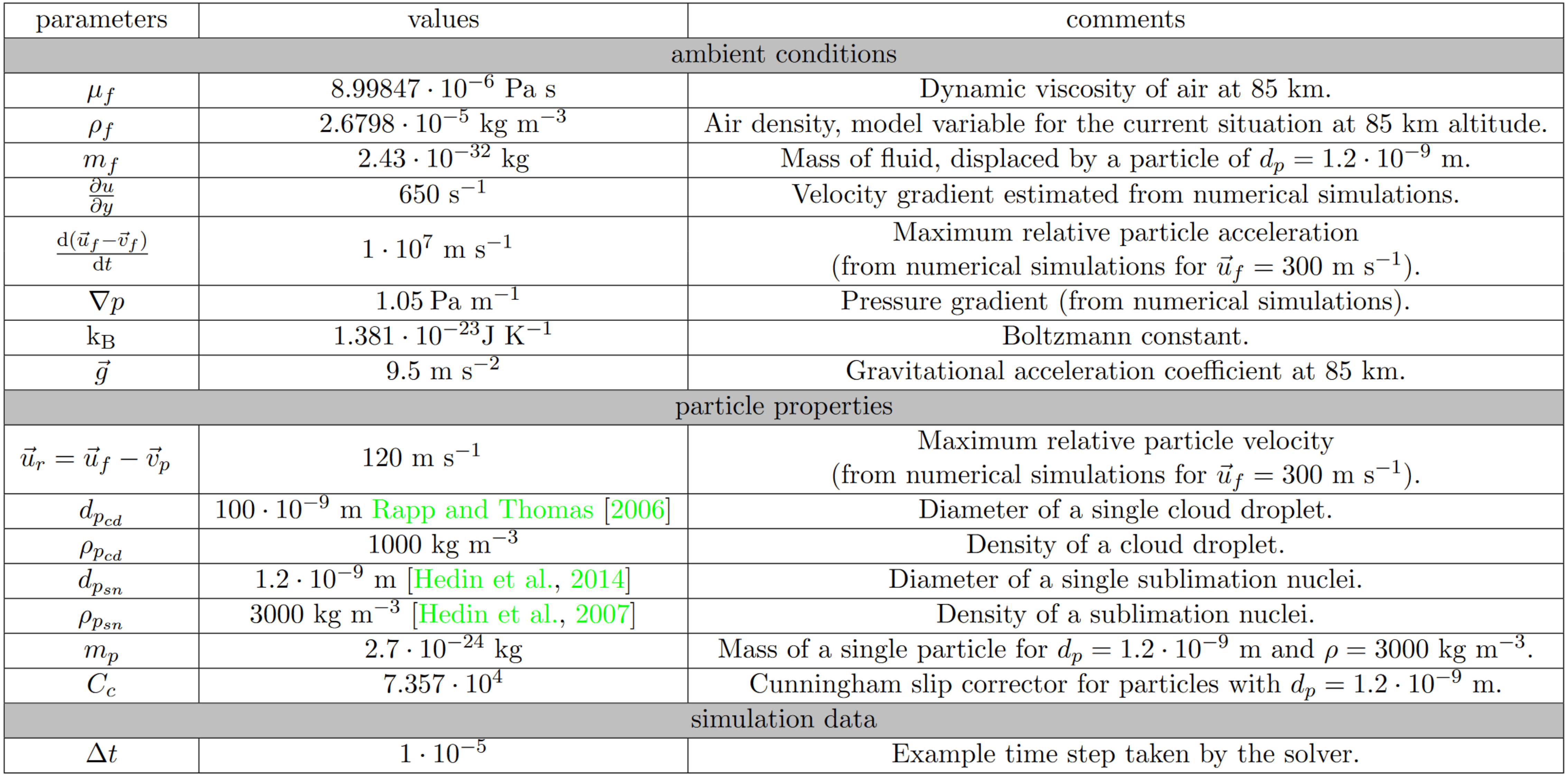
Turco et al., Planetary and Space Science, 30:1147-1181, 1982.

MSP particle

$\rho_{MSP} = 3\ \dfrac{\mathrm{g}}{\mathrm{cm}^3}$

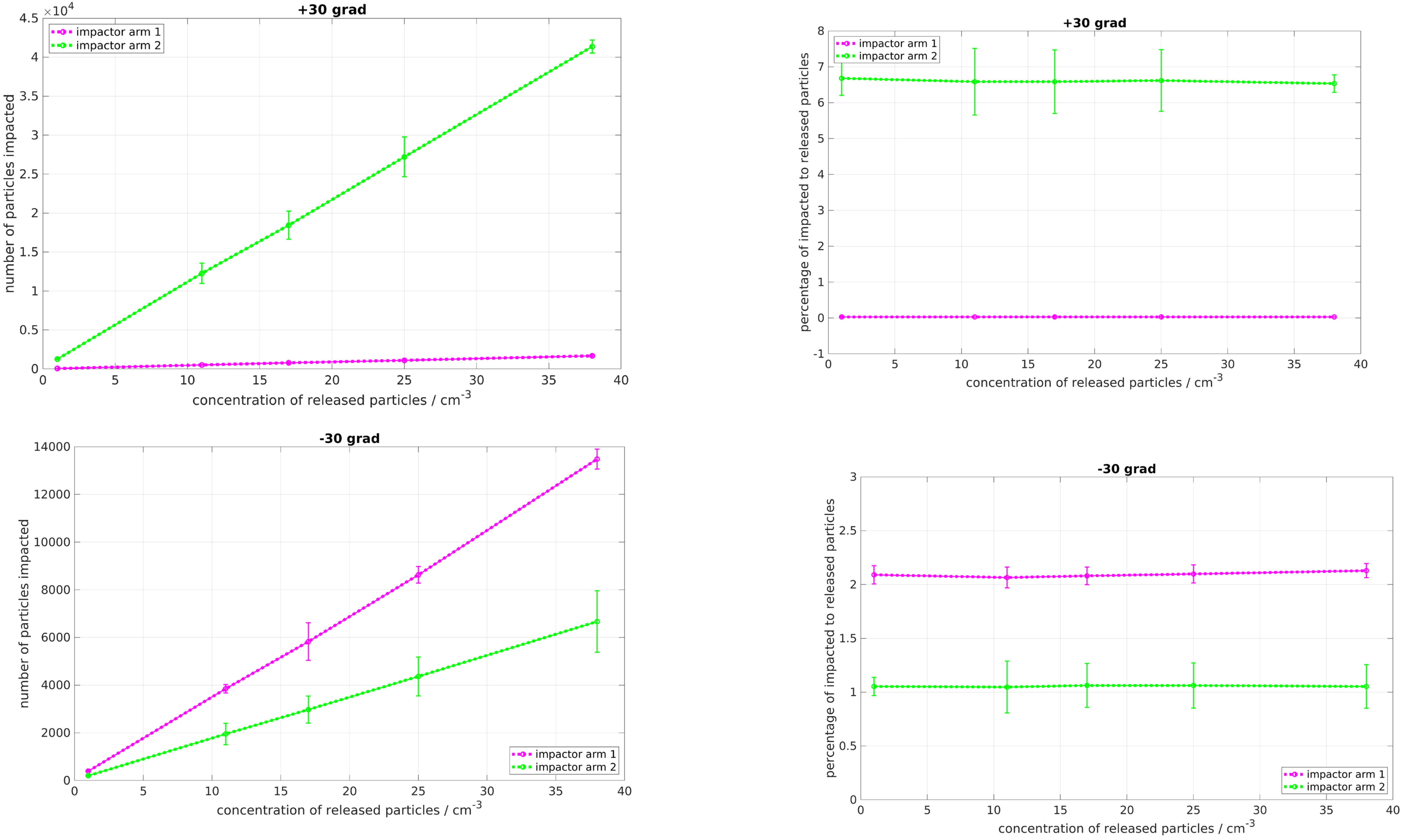
$r_{MSP} = 0.4 - 0.8\ \mathrm{nm}$

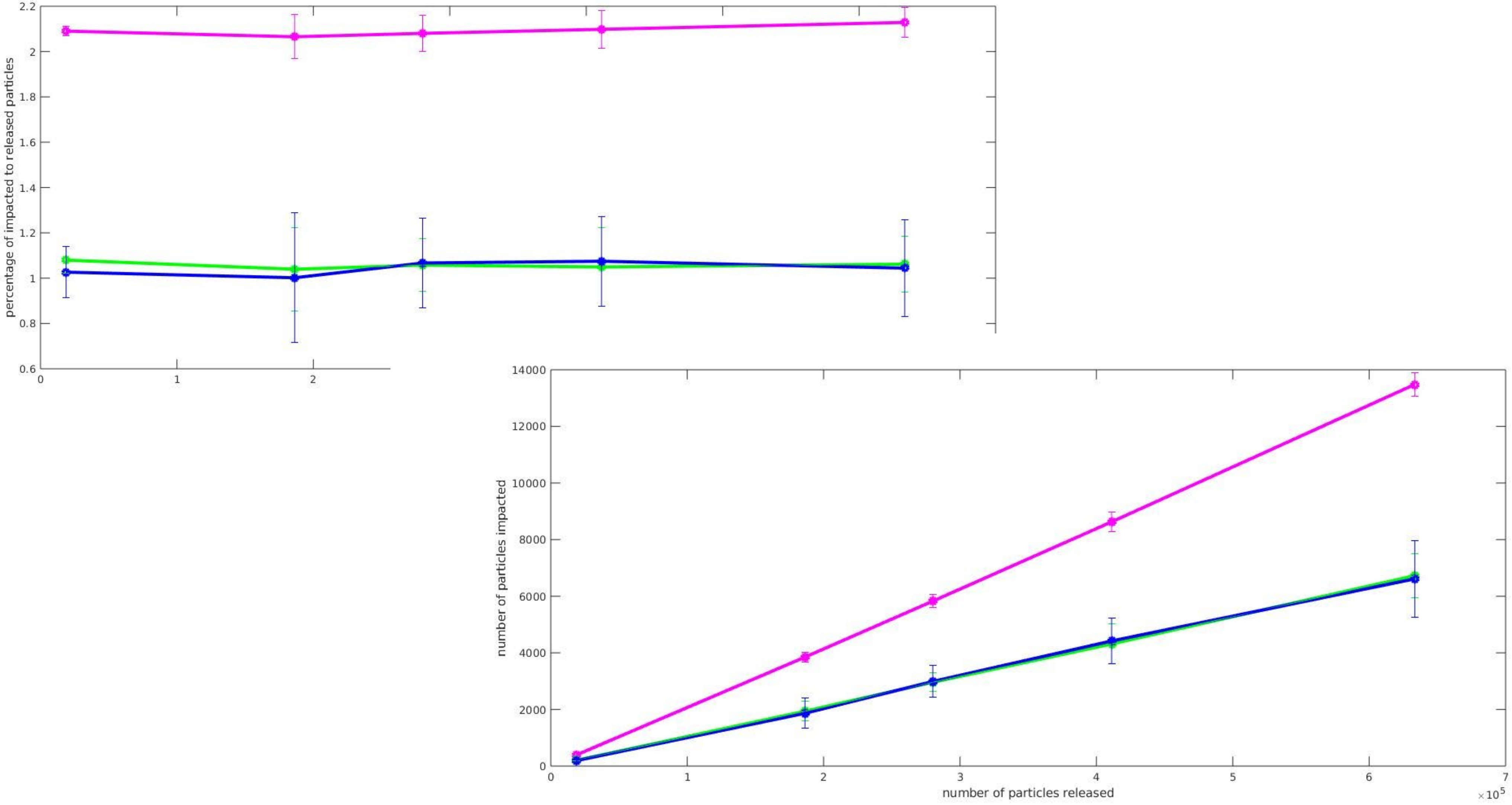
Hedin et al., Journal of Atmospheric and Solar-Terrestrial Physics, 118:127-114, 2014.



parameters	values	comments
ambient conditions		
μ_f	$8.99847 \cdot 10^{-6}$ Pa s	Dynamic viscosity of air at 85 km.
ρ_f	$2.6798 \cdot 10^{-5}$ kg m^{-3}	Air density, model variable for the current situation at 85 km altitude.
m_f	$2.43 \cdot 10^{-32}$ kg	Mass of fluid, displaced by a particle of $d_p = 1.2 \cdot 10^{-9}$ m.
$\frac{\partial u}{\partial y}$	650 s^{-1}	Velocity gradient estimated from numerical simulations.
$\frac{\mathrm{d}(\vec{u}_f - \vec{v}_f)}{\mathrm{d}t}$	$1 \cdot 10^{7}$ m s^{-1}	Maximum relative particle acceleration (from numerical simulations for $\vec{u}_f = 300$ m s^{-1}).
∇p	1.05 Pa m^{-1}	Pressure gradient (from numerical simulations).
$\mathrm{k_B}$	$1.381 \cdot 10^{-23}$J K^{-1}	Boltzmann constant.
$\vec{g}$	9.5 m s^{-2}	Gravitational acceleration coefficient at 85 km.
particle properties		
$\vec{u}_r = \vec{u}_f - \vec{v}_p$	120 m s^{-1}	Maximum relative particle velocity (from numerical simulations for $\vec{u}_f = 300$ m s^{-1}).
$d_{p_{cd}}$	$100 \cdot 10^{-9}$ m Rapp and Thomas [2006]	Diameter of a single cloud droplet.
$\rho_{p_{cd}}$	1000 kg m^{-3}	Density of a cloud droplet.
$d_{p_{sn}}$	$1.2 \cdot 10^{-9}$ m [Hedin et al., 2014]	Diameter of a single sublimation nuclei.
$\rho_{p_{sn}}$	3000 kg m^{-3} [Hedin et al., 2007]	Density of a sublimation nuclei.
m_p	$2.7 \cdot 10^{-24}$ kg	Mass of a single particle for $d_p = 1.2 \cdot 10^{-9}$ m and $\rho = 3000$ kg m^{-3}.
C_c	$7.357 \cdot 10^{4}$	Cunningham slip corrector for particles with $d_p = 1.2 \cdot 10^{-9}$ m.
simulation data		
Δt	$1 \cdot 10^{-5}$	Example time step taken by the solver.

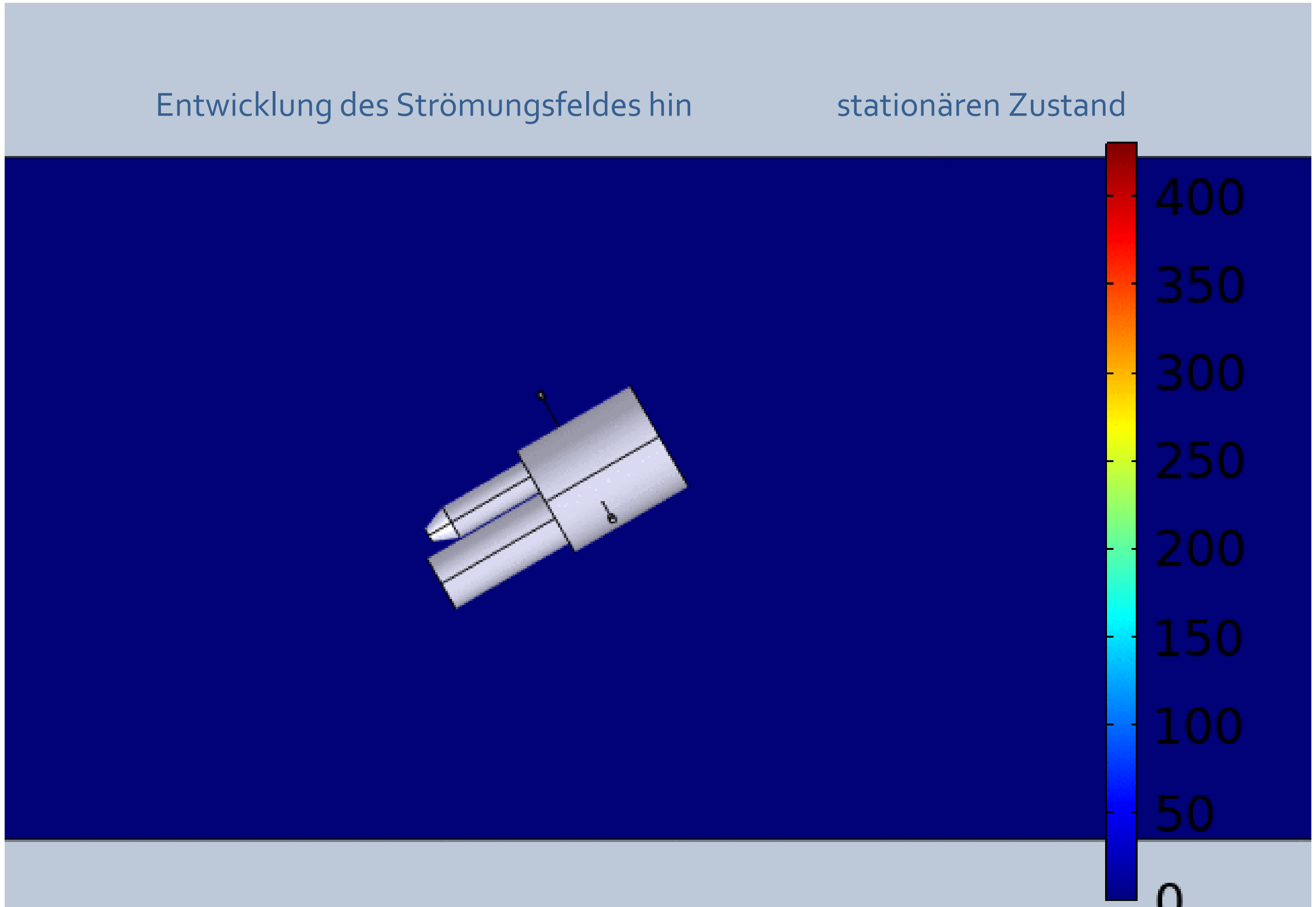
Generalized alpha methode


+30 grad
×10^4
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
number of particles impacted
impactor arm 1
impactor arm 2
0
5
10
15
20
25
30
35
40
concentration of released particles / cm^-3
+30 grad
8
7
6
5
4
3
2
1
0
-1
percentage of impacted to released particles
impactor arm 1
impactor arm 2
0
5
10
15
20
25
30
35
40
concentration of released particles / cm^-3
-30 grad
14000
12000
10000
8000
6000
4000
2000
0
number of particles impacted
impactor arm 1
impactor arm 2
0
5
10
15
20
25
30
35
40
concentration of released particles / cm^-3
-30 grad
3
2.5
2
1.5
1
0.5
0
percentage of impacted to released particles
impactor arm 1
impactor arm 2
0
5
10
15
20
25
30
35
40
concentration of released particles / cm^-3


percentage of impacted to released particles
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
1
2
number of particles impacted
14000
12000
10000
8000
6000
4000
2000
0
0
1
2
3
4
5
6
7
$\times 10^5$
number of particles released

GEL Projekt

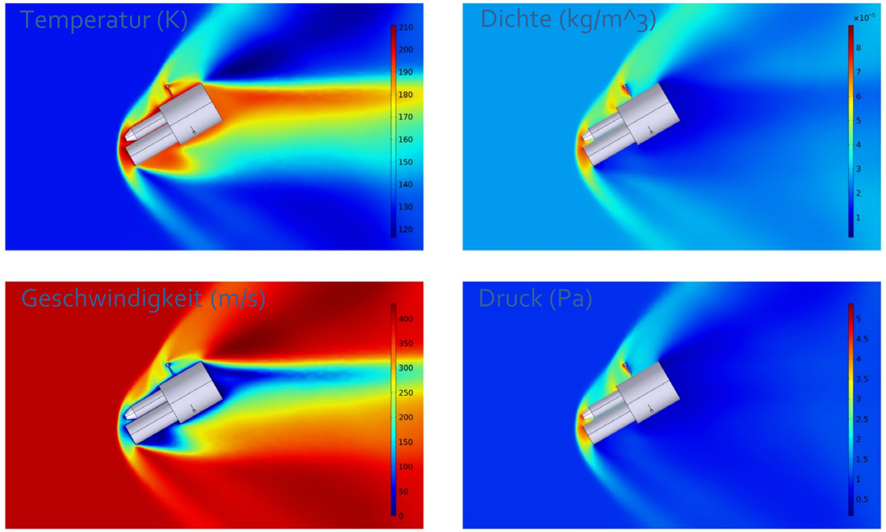
Simulation einer **supersonischen Strömung** um eine Höhenforschungsrakete und die **Sammeleffizienz von mesosphärischen Partikeln** auf Impaktorflächen



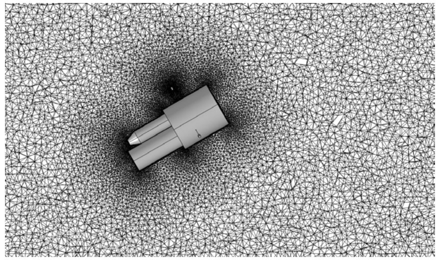
Entwicklung des Strömungsfeldes hin stationären Zustand

$$Ma = \frac{u}{c} = 1.75$$

$$c_{85\ \mathrm{km}} = \sqrt{\gamma R_s T} = 229\ \frac{\mathrm{m}}{\mathrm{s}}$$



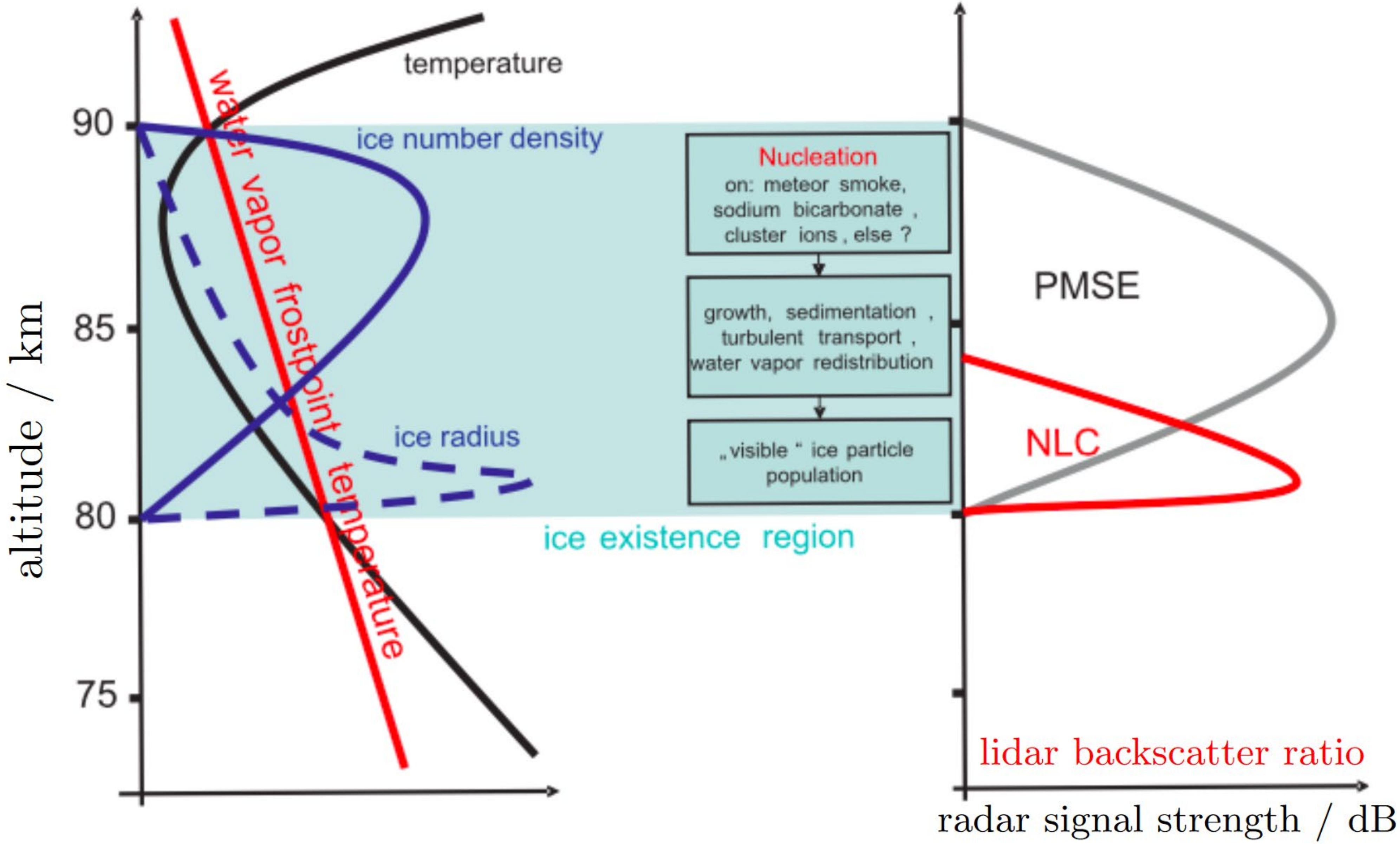
Stationäres Strömungsfeld



Berechnungsnetz

$$Ma = \frac{u}{c} = 1.75$$

Meteoroid → Small (sub-km) rocky or metallic body in outer space.
Meteor → Light phenomenon ("shooting star" or "falling star“), visible passage of a frictionally heated and glowing body from outer space.
Meteorite → Solid piece or debris from outer space which has survived the passage through the atmosphere and has hit the surface → **Micrometeorite** if $D_p < 1$ mm.



Nucleation
on: meteor smoke, sodium bicarbonate , cluster ions , else ?

growth, sedimentation , turbulent transport , water vapor redistribution

„visible " ice particle population

forces and their relationship R to the Stokes drag force	importance in reference to Stokes drag force
pressure gradient force: $R_p = \left\lvert \frac{\vec{F}_p}{\vec{F}_D} \right\rvert \sim \frac{d_p^2 \nabla p}{\vec{u}_r}$	Neglectable for nanometre-sizes of d_p and small pressure gradients around the particle.
gravitational force: $R_{G,tot} = \left\lvert \frac{\vec{F}_g}{\vec{F}_D} \right\rvert \sim \frac{d_p^2}{\vec{u}_r}$	Neglectable for nanometre-sized particles.
Saffman force: $R_L = \left\lvert \frac{\vec{F}_L}{\vec{F}_D} \right\rvert \sim d_p \sqrt{\frac{\vec{u}_r \times [\nabla \times \vec{u}_r]}{\vec{u}_r}}$	Neglectable for nano size particles or small relative velocities.
added mass force: $R_{am} = \left\lvert \frac{\vec{F}_{am}}{\vec{F}_D} \right\rvert \sim d_p^2 \frac{\mathrm{d}(\vec{u}_f - \vec{v}_p)}{\mathrm{d}t}$	Small for very small particles and for a small relative acceleration of the particle.
Brownian force: $R_{Brown} = \left\lvert \frac{\vec{F}_{Brown}}{\vec{F}_D} \right\rvert \sim \sqrt{\frac{1}{d_p \vec{u}_r^2}}$	Indispensable for nanometre-sized particles.

$$Ma = \frac{|v|}{c}, \qquad c = \sqrt{\gamma \ \mathrm{R_s} \ T},$$

$$c_{85\ \mathrm{km}} = 229\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$v_{\mathrm{min}} = 300\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$v_{\mathrm{max}} = 400\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$Ma_{\mathrm{min}} = 1.31$$

$$Ma_{\mathrm{max}} = 1.75$$

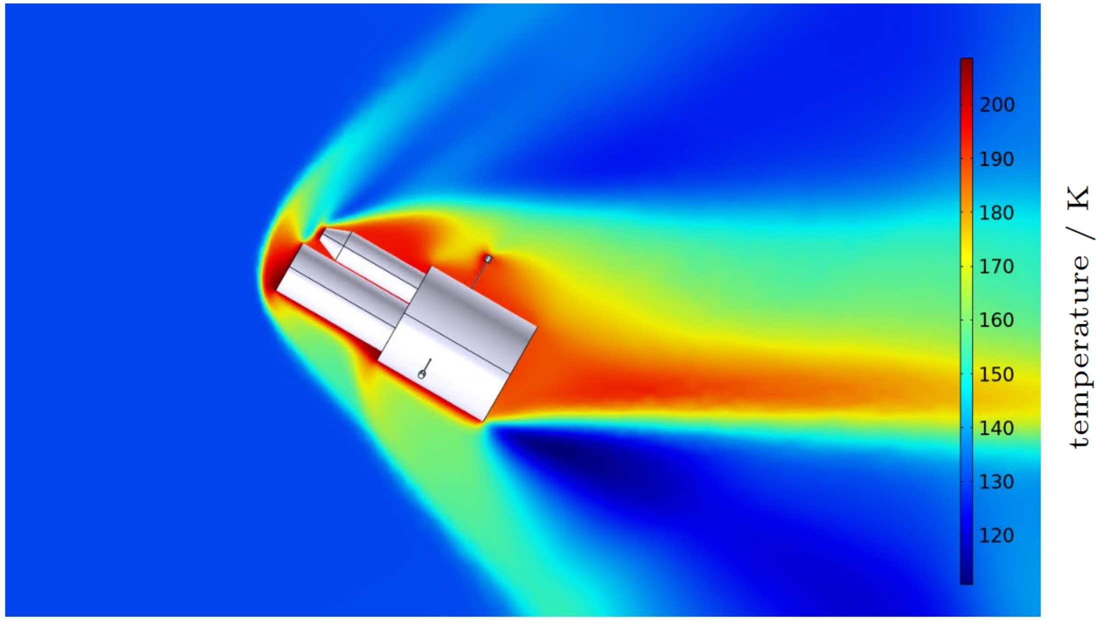
$$Ma = \frac{|v|}{c}, \qquad c = \sqrt{\gamma \, \mathrm{R_s} \, T},$$

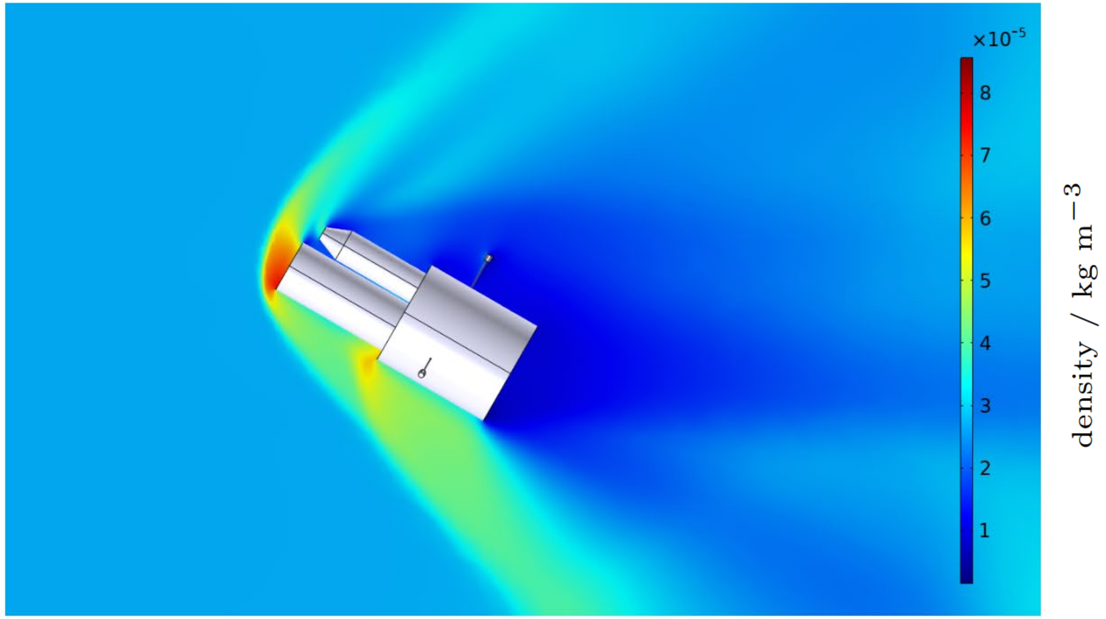
$$\underbrace{\frac{\partial \rho}{\partial t}}_{\substack{\text{temporal change}\\ \text{of mass}}} + \underbrace{\nabla \cdot (\rho \, \vec{u})}_{\substack{\text{change}\\ \text{of mass flux}}} = 0.$$

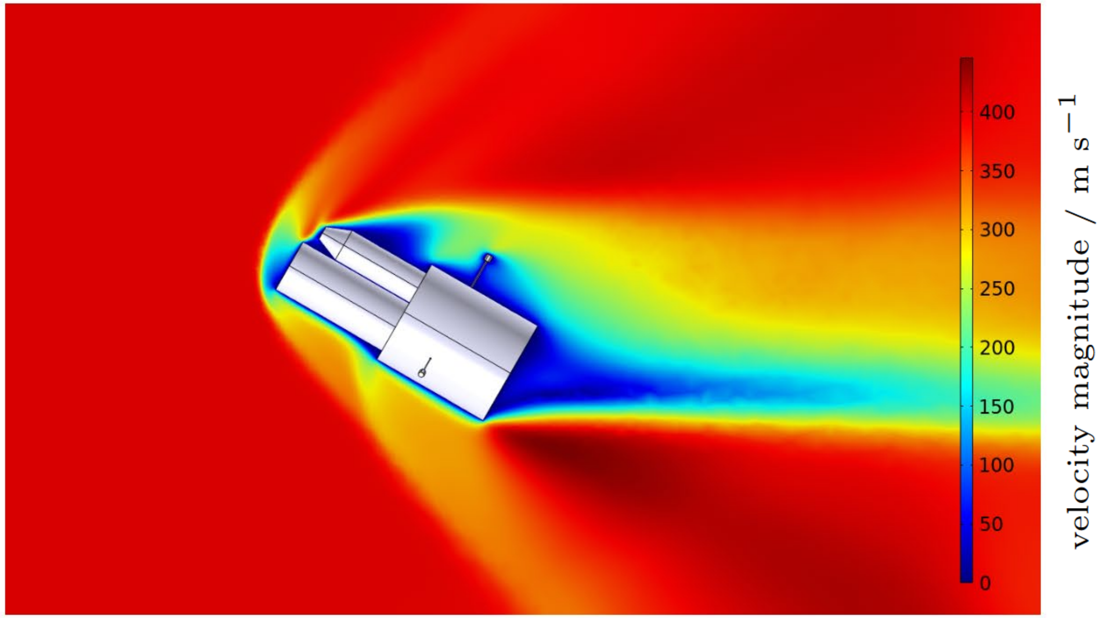
$$\underbrace{\rho \frac{\partial \vec{u}}{\partial t}}_{\substack{\text{temporal change}\\ \text{of momentum}}} + \underbrace{\rho(\vec{u} \cdot \nabla)\vec{u}}_{\substack{\text{change of}\\ \text{momentum by advection}}} = \underbrace{-\nabla p \mathbf{I}}_{\substack{\text{force from}\\ \text{pressure gradient}}} + \underbrace{\mu \Delta \vec{u}}_{\substack{\text{viscous shear}\\ \text{surface force}}} + \underbrace{\rho \vec{f}.}_{\text{body force}}$$

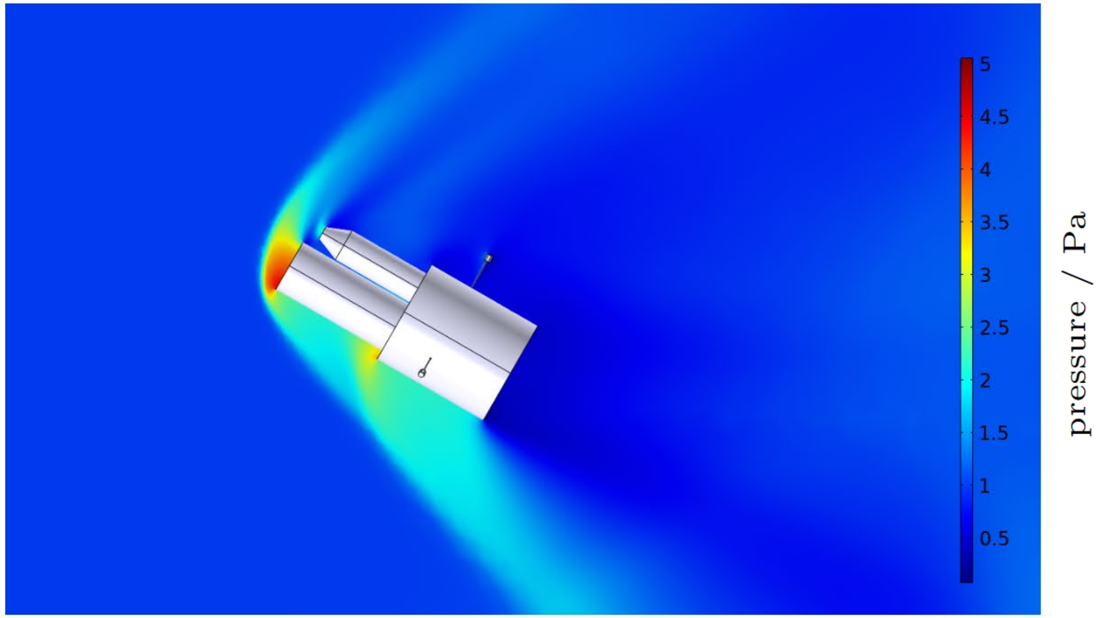
$$\underbrace{\frac{\partial}{\partial t}\left[\rho\left(e + \frac{u^2}{2}\right)\right]}_{\substack{\text{temporal change}\\ \text{of total energy}}} + \underbrace{\nabla \cdot \left[\rho \vec{u}\left(e + \frac{u^2}{2}\right)\right]}_{\substack{\text{net rate of flow}\\ \text{of total energy}}} = \underbrace{\nabla \cdot \vec{q}}_{\text{heat addition}} + \underbrace{\nabla \cdot (\sigma \vec{u})}_{\substack{\text{work due to}\\ \text{pressure force}}} + \underbrace{\rho(\vec{f} \cdot \vec{u}).}_{\substack{\text{work due to}\\ \text{body force}}}$$

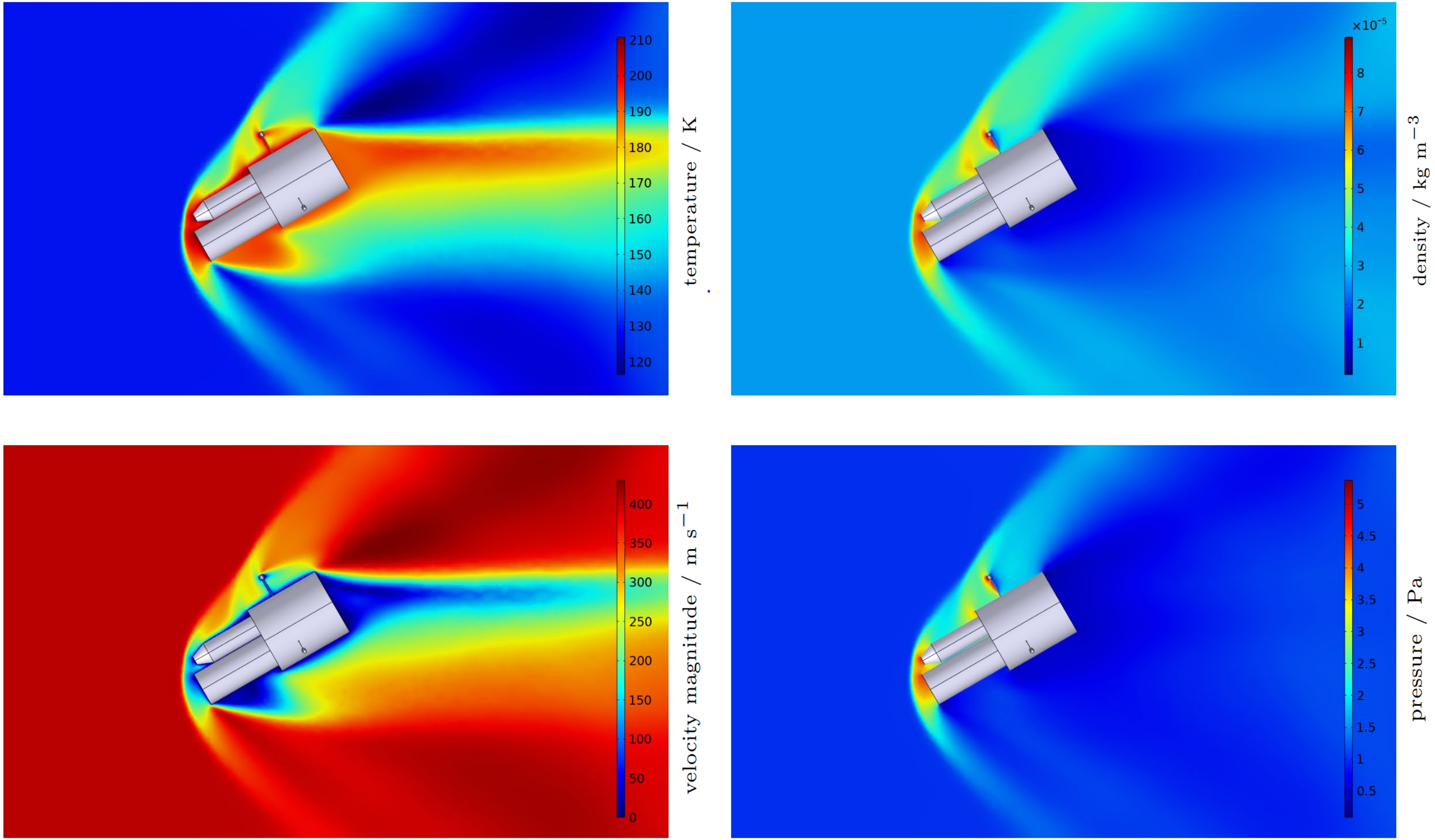
$$e = c_v \, T, \qquad \rho = \frac{p}{\mathrm{R_s} \, T},$$


200
190
180
170
160
150
140
130
120
temperature / K


×10^{-5}
8
7
6
5
4
3
2
1
density / kg m^{-3}

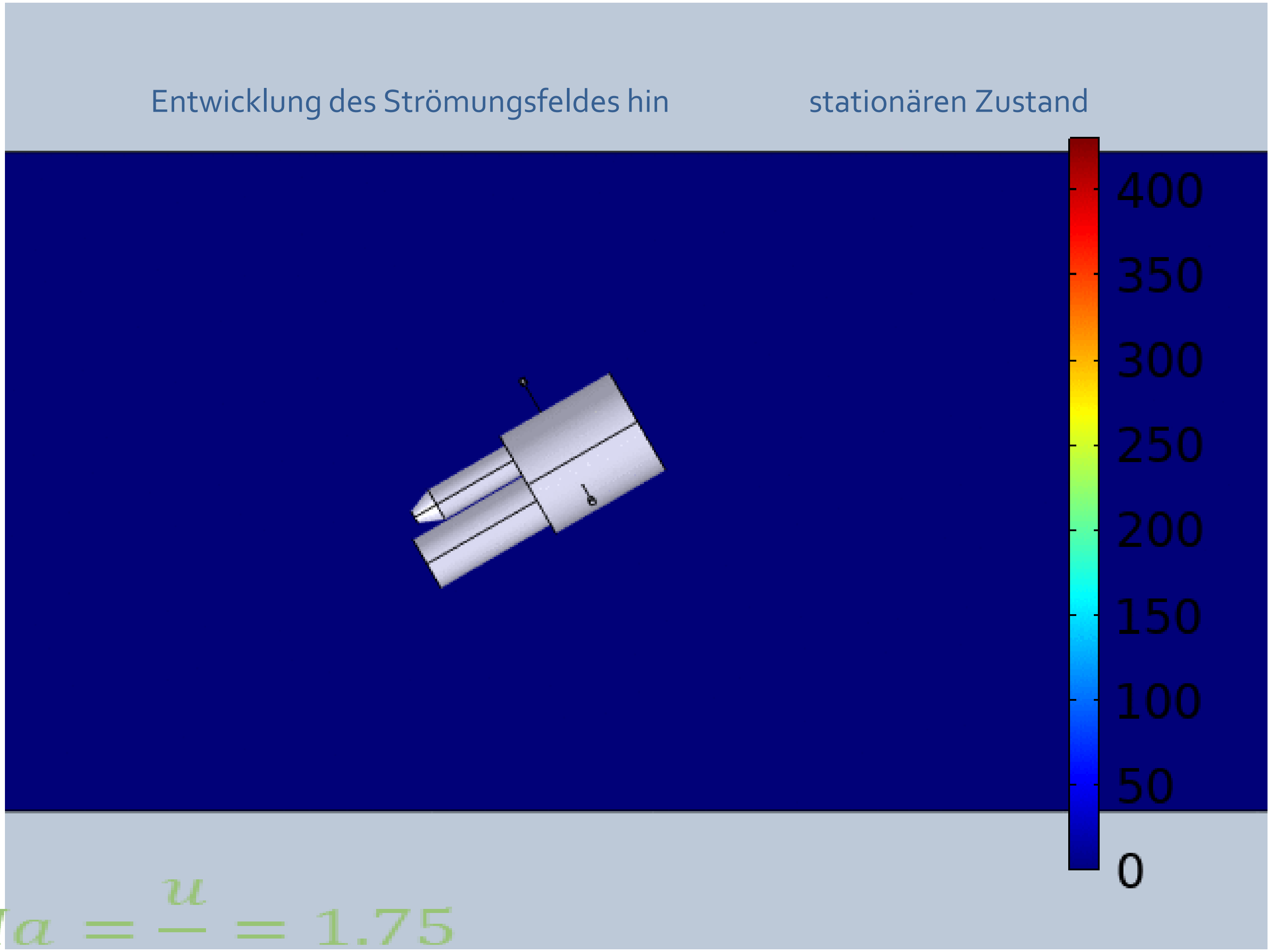

400
350
300
250
200
150
100
50
0
velocity magnitude / m s^{-1}


5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
pressure / Pa


210
200
190
180
170
160
150
140
130
120
temperature / K
×10⁻⁵
8
7
6
5
4
3
2
1
density / kg m⁻³
400
350
300
250
200
150
100
50
0
velocity magnitude / m s⁻¹
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
pressure / Pa

GEL Projekt

Simulation einer **supersonischen Strömung** um eine Höhenforschungsrakete



$$Ma = \frac{u}{c} = 1.75$$

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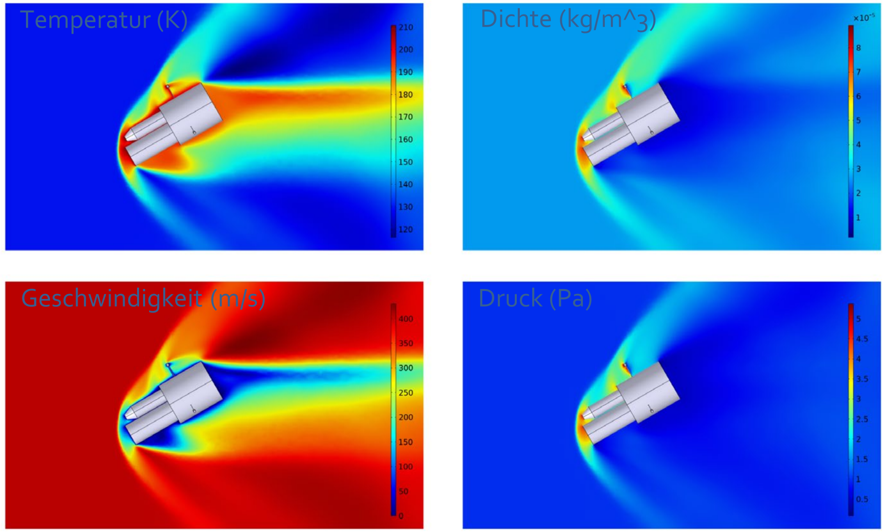
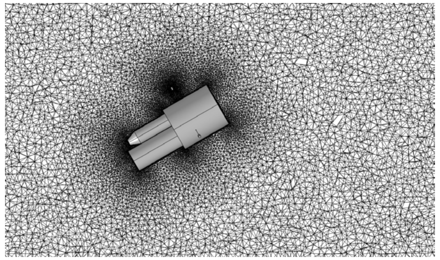
$$c_{85\ \mathrm{km}} = \sqrt{\gamma R_s T} = 229\ \frac{\mathrm{m}}{\mathrm{s}}$$



Stationäres Strömungsfeld



Berechnungs-
netz