The effect of viscosity on dynamics of Electro-wetting droplet

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Electrowetting (EW) has found a wide range of the potential applications including fast response displays, lab-on-a-chip microfluidic devices, and light vales. However, fundamental understanding of dynamics of EW-actuated droplet is not clear. In this paper, a numerical method was used to investigate the Electro-wetting response of a droplet subject to direct current (DC) actuating signals. The moving mesh method was used to track gas-liquid interface. A dynamic contact angle (DCA) model with molecular kinetic theory (MKT-DCA) was set as the boundary condition. The MKT-DCA model takes the effects of the electrical force, the contact line friction and the pinning force into account at the contact line. The different viscosity droplet shape evolution under DC condition were studied. It was proved that the computational fluid dynamics models (CFD) can accurately predict dynamics of EW-induced droplet.

Keywords: Droplet electro-wetting, moving mesh method, viscosity, dynamics

1. Introduction

Droplet dynamics (spreading and oscillation) on a solid substrate has attracted great interest due to its various applications, such as fast response displays, lab-on-a-chip microfluidic devices, and light vales, etc. Among all means to control the droplet motion, EW is particularly convenient due to its ability to change contact angle with an electric field. With EW, it is possible to design digitized cooling devices for thermal management of compact microsystems[1].

EW can change the apparent contact angle due to the electrical force concentrated at the three-phase contact line (TCL). The variation of apparent contact angle by EW can be described by the Lippmann equation. In the past, both the hydrodynamic theory[2] and the molecular kinetic theory (MKT)[3] have been used to model the DCA of a wetting droplet on a solid surface. The hydrodynamic theory attributes the deviation of the dynamic contact angle from static contact angle to bulk viscous dissipation near the moving contact line. The MKT takes into account the friction force at contact line. Former literatures suggest the MKT is more suitable for behavior of EW because it offers a explanation of the contact angle hysteresis phenomenon. Recently, some numerical and experimental studies have demonstrated the validity of MKT in EW applications. Annapragada et al.[4] studied numerically the EW response of a droplet on the substrate to a DC signal with a MKT-based DCA model, and formulated a kinematic equation which correctly predicted the overall trend of the contact line motion of droplet .

The dynamic process of droplet Electro-wetting is complex and its basic mechanism is not fully understood. This work is aimed to investigate the dynamics of Electro-wetting droplet on the solid surface using COMSOL moving mesh method. The established droplet electro-wetting model will be further coupled with heat transfer and phase change models for thermal management of compact microsystems application.

2. Governing equations

The fluid flow of liquid droplet is incompressible and laminar. The density and viscosity are constant for both air and liquid droplet.

The mass conservation equation is:

$$\nabla u = 0 \tag{1}$$

The momentum conservation equation is:

$$\rho\left(\frac{\partial u}{\partial t} + (u - u_m)\nabla u\right) = -\nabla p + \nabla\left(\mu\left(\nabla u + \left(\nabla u\right)^T\right)\right) + \rho g + F_{st} \tag{2}$$

Where ρ is the fluid density, u is the velocity field, u_m stands for the mesh velocity, p stands for the pressure in the flow field, μ stands for the fluid dynamic viscosity, g is the gravitational acceleration, F_{st} is the surface tension force.

The interface are tracked using the moving mesh method. In the absence of mass flow across the boundary, the correct boundary condition on the air/liquid interface is

$$n \cdot \left(\mathrm{T}_{air} - \mathrm{T}_{liquid}\right) = \gamma_{LV}\,(\nabla \cdot n)\,n - \nabla \gamma_{LV} \tag{3}$$

Where n is the unit vector normal to interface, T_{air} is the total stress tensor for the air, T_{liquid} is the total stress tensor for the liquid, γ_{LV} is surface tension coefficient. T_{air} is defined as

$$T_{air} = -p_{air} I + \mu_{air}\left(\nabla u_{air} + \left(\nabla u_{air}\right)^T\right) \tag{4}$$

T_{liquid} is defined as

$$T_{liquid} = -p_{liquid} I + \mu_{liquid}\left(\nabla u_{liquid} + \left(\nabla u_{liquid}\right)^T\right) \tag{5}$$

A mesh velocity equal to the fluid velocity is imposed on the interface:

$$u_m = u \tag{6}$$

In the electro-wetting, the contact angle change is made by the induced electrical force. The forces in the three phase contact line are balanced[5].

$$\gamma_{LV}\cos\theta_D = F_c + F_f + F_{el} \tag{7}$$

The electrical force (F_{el}) is

$$F_{el} = \frac{1}{2}\frac{\varepsilon_0\varepsilon_r}{d}V^2 \tag{8}$$

Here ε_0 stands for the vacuum permittivity constant, ε_r is the relative permittivity of the insulating dielectric layer, d is the thickness of the dielectric layer, and V stands for the applied voltage.

The capillary force (F_c) is

$$F_c = \gamma_{LV}\cos\theta_e \tag{9}$$

Considering the pinned edge effect, the contact line friction force (F_f) is

$$F_f = -\left[\varsigma U + c_{pin}\,\mathrm{sgn}(U) - \frac{c_{pin}}{\pi/2}\tan^{-1}\left(\frac{\varsigma U}{c_{pin}/(\pi/2)}\right)\right] \tag{10}$$

Here ζ is the coefficient of the three phase contact line friction, c_{pin} is the contact angle hysteresis effect, U is the velocity of the three phase contact line motion.

The following dynamic contact angle (DCA) model is developed

$$\theta_D = \cos^{-1}\left\{\left[\left[\cos\theta_e - \left[\varsigma U + c_{pin}\,\mathrm{sgn}(U) - \frac{c_{pin}}{\pi/2}\tan^{-1}\left(\frac{\varsigma U}{c_{pin}/(\pi/2)}\right)\right]\right]/\gamma_{LV} + \frac{1}{2}\frac{\varepsilon_0\varepsilon_r}{d\gamma_{LV}}V^2\right]\right\} \tag{11}$$

Finally, the DCA model is applied as a boundary condition in this simulations.

3. Numerical model

The numerical model is implemented in COMSOL 5.4 using the laminar flow, two phase flow and moving mesh interface. The two-dimensional axis-symmetric domain is shown in Figure 1.

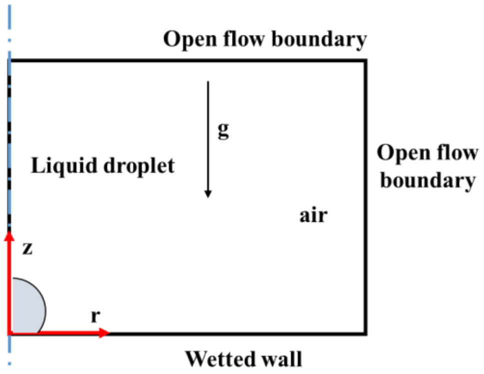


Figure 1. Schematic of computational domain

The top and right side of the computational domain are open boundary conditions. The substrate is wetted boundary. The velocity component normal to the wall is zero. The boundary condition also allows specifying the dynamic contact angle θ_D between the wall and the fluid interface. The maximum grid size is 0.135mm, the minimum grid size is 0.001mm, mesh growth rate is 1.15 and curvature factor is 0.3. The relative tolerance is 0.001 and Time step is 0.001s.

As shown in Figure 2, dynamic contact angle θ_D affects the droplet wet radius R and height h during the droplet electro-wetting process.

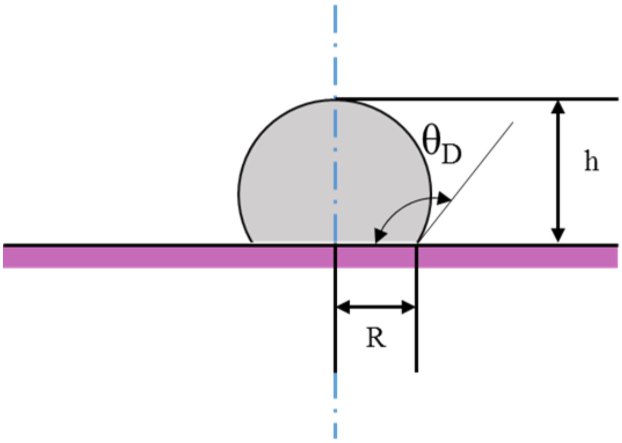


Figure 2. Schematic of electro-wetting droplet on a surface

The properties of droplet and air, dielectric layer, Hydrophobic layer and voltage are listed in Table 1 and 2.

Table 1. Properties of liquid and air

Parameter	Symbol	Value	Unit

Density of liquid	ρ_l	1000	Kg/m^3
Dynamics viscosity of liquid	μ_l	0.0001,0.001,0.01,0.1,0.8	Pa·s
Density of air	ρ_a	1.204	Kg/m^3
Dynamics viscosity of air	μ_a	$1.814*10^{-5}$	Pa·s
Surface tension	σ	0.0712	N/m
Droplet diameter	D	1.016	mm
Initial contact Angle	Θ_e	118	°

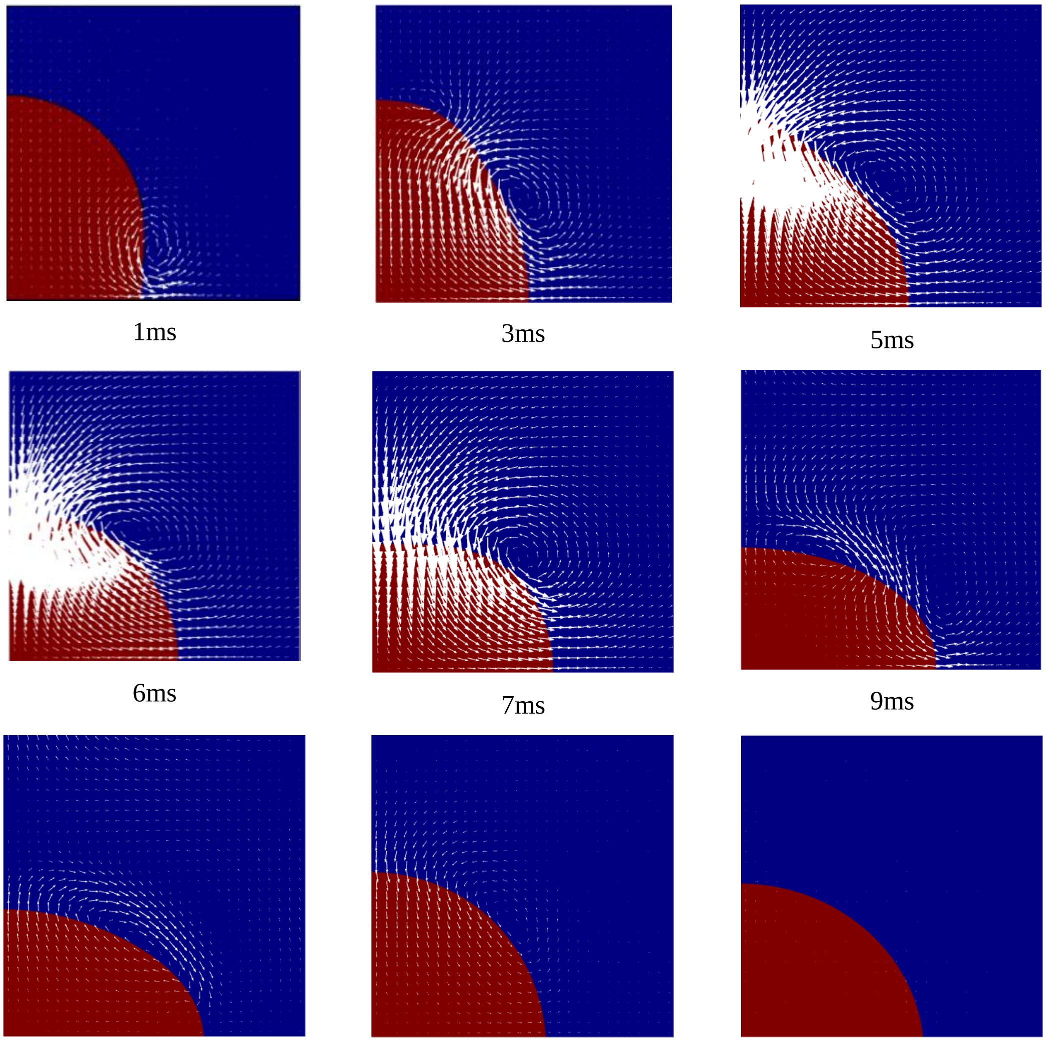
Table 2. Properties of dielectric layer and Hydrophobic layer

Parameter	Symbol	Value	Unit
Thickness of dielectric layer	d	5	um
Relative permittivity of the insulating dielectric layer	ε_r	3.15	
Hydrophobic layer(PTFE)	d_h	100	nm

4. Results

The paper presented simulation results of droplet with different viscosity on the substrate by DC electro-wetting.

Fig. 3 shows simulation predictions of the instantaneous droplet ($\mu_{l,}$ =0.001 Pa·s) shape at different time for input voltage (V =120 V). Once the input voltage is loaded, the contact line moves outward. The shape of the droplet changes because the electrical force reduces the apparent contact angle. Velocity vectors are overlaid on the droplet contours. The high velocity region represents the starting of surface waves, and the rest region of droplet remains stationary. As time increases, the surface waves propagate upward along the air-liquid interface, causing the droplet to distort continuously. Subsequently, the capillary waves are flashed back to the bottom of the droplet, thereby lowering the height of droplet and spreading the contact radius of droplet on the wall. Finally, it undergoes damped oscillations until a new equilibrium is established.


1ms 3ms 5ms

6ms 7ms 9ms

10ms 20ms 50ms

Figure 3. Electro-wetting process of droplet on the substrate (V=120V, $\mu_{l,}$ =0.001 Pa·s)

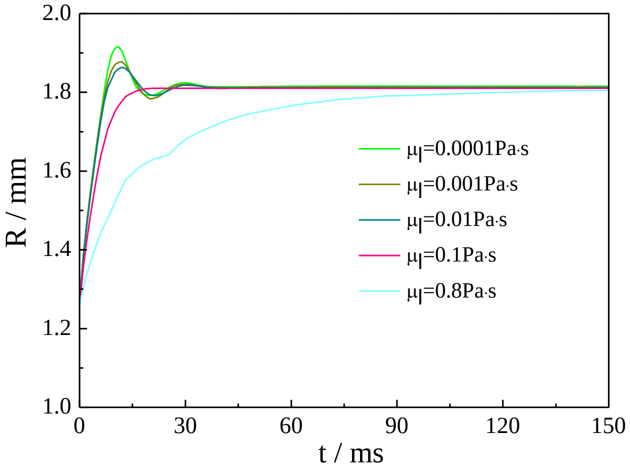


Figure 4. Time evolution of contact radius of droplet with different viscosity in DC (V=120V)

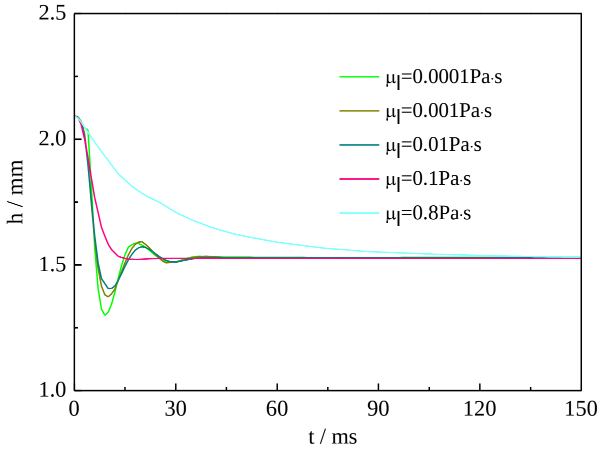


Figure 5. Time evolution of height of droplet with different viscosity in DC (*V*=120V)

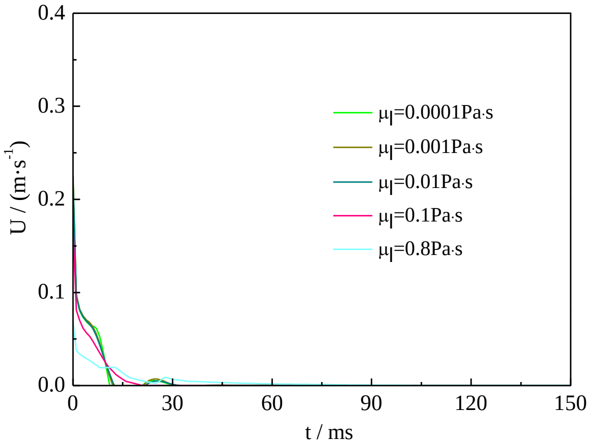


Figure 6. Time evolution of contact line velocity of droplet with different viscosity in DC (*V*=120V)

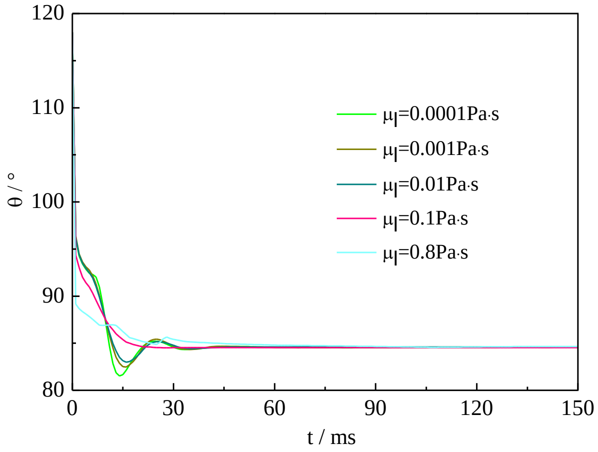


Figure 7. Time evolution of contact angle of droplet with different viscosity in DC (*V*=120V)

The time evolution of the wetting radius, height, contact line velocity and dynamic contact angle of the droplet with different viscosity are plotted for input voltage (*V* = 120 V) in Fig. 4-7. The results show that when the viscosity is low(0.01Pa·s), the contact line first expands on the wall and, after reaching the maximum radius, minimum height, minimum contact line velocity and minimum dynamic contact angle at t = 11 ms, it undergoes damped oscillations until a new equilibrium is established at t = 15-20 ms. The oscillations in contact radius is the result of internal flow induced by the contact line motion. When liquid viscosity is high (0.1-0.8Pa·s), the contact line first expands on the wall, radius reach maximum, and height, contact line velocity and dynamic contact angle reach minimum, Oscillations don’t occurs.

5. Conclusions

A numerical approach was used to explore the dynamics of EW droplet. The time evolution of the contact radius, height, three phase contact line velocity and dynamic contact angle of the droplet was investigated in the DC signal. The moving mesh method is applied to develop CFD models coupled with an MKT-based dynamic contact angle model. This study will help to advance the fundamental understanding of EW droplet dynamics. Some findings can be summarized as follows: (1) The MKT based dynamic contact angle model can be successfully used to numerical analysis of droplet dynamics. (2) When droplet’s viscosity is low, it recoils and undergoes damped oscillations under DC Electro-wetting. When droplet’s viscosity is high, oscillations don’t occurs.

6. References

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