

Using COMSOL for the Development of the UK's Second Hyperloop Prototype

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Abstract

Hyperloop is a concept proposed by Elon Musk back in 2012 for the future of mass transportation. It uses electromagnetic (EM) propulsion to accelerate passenger or cargo pods through a low-pressure environment. The UK's second prototype that uses such propulsion and levitation systems was developed by HYPED (The University of Edinburgh's Hyperloop Team) and used in the 2018 SpaceX Hyperloop Pod Competition. Various simulations were required to determine the design parameters for the systems used in the prototype. In this paper, a linear Halbach array and electrodynamic wheels (EDWs) to confirm the design parameters chosen.

1 Introduction

The Hyperloop concept relies on an autonomous pod traveling through a low-pressure tube at speeds potentially comparable to that of a commercial airliner. This could mean slashing travel times for intercity travel and completely revolutionise the urban landscape. As such, there has been a significant interest in this technology from different parties (governments and corporations alike) in the past couple of years. One of the interested parties is SpaceX, which hosts a student competition aiming to build the fastest pod each year. The University of Edinburgh's Hyperloop team (HYPED) developed such a prototype and placed 6th in the 2018 competition.

The pod is propelled and levitated electromagnetically using so-called Electrodynamic Wheels (EDWs) and Halbach Levitation Arrays respectively. Therefore, determining correct the design parameters to get the performance required from these systems was an essential part of the design process for the prototype pod. Furthermore, as the track heats up due to eddy currents, relevant thermal simulations were performed. In order to achieve this, several COMSOL modules were used. In this paper, we discuss our simulation setup in Section 3. We then present our results in Section 4 and discuss them and compare them to a numerical model.

2 Theory

2.1 Halbach Levitation Arrays

The pod was equipped with 4 arrays containing magnets arranged in a Halbach configuration [Figure 1].

The general shape of the Halbach levitation arrays' magnetic field was derived by J.C. Mallinson [1], which is simply the superposition of two trigonometry functions and shown as [Figure 6].



Figure 1: Halbach Array Magnetic Field Orientation

As the pod travels above a conductive non-magnetic track eddy currents are generated by the time varying magnetic field. According to Lenz's law, these currents generate an opposing magnetic field, thus producing a force which lifts pods above the ground. Note that due to sinusoidal shape of the original field, a drag force is also produced. The equations describing the lift and drag forces of the Halbach Arrays are given by:

$$F_y = F_y^{max} \frac{\left(\sqrt{1 + \frac{\kappa^4 \delta^4}{4}} - \frac{\kappa^2 \delta^2}{2}\right)^{\frac{3}{2}}}{\kappa \delta + \left(\sqrt{1 + \frac{\kappa^4 \delta^4}{4}} + \frac{\kappa^2 \delta^2}{2}\right)^{\frac{3}{2}}} \quad (1)$$

$$F_z = F_y \frac{\kappa \delta}{\left(\sqrt{1 + \frac{\kappa^4 \delta^4}{4}} - \frac{\kappa^2 \delta^2}{2}\right)^{\frac{1}{2}}} \quad (2)$$

Where,

$$F_y^{max} = \frac{B_0^2 A}{\mu_0} e^{-2\kappa y_1 - \kappa \Delta_c} \quad (3)$$

$$\delta = \sqrt{\frac{2\rho}{\mu_0 \kappa v}} \quad (4)$$

With:

- B_0 : magnetic field amplitude of the magnets
- A : total area of levitation array
- κ : wave number of the Halbach array
- μ_0 : permeability of free space
- Δ_c : thickness of the track
- y_1 : air gap between the track and lower side of Halbach array
- ρ : conductivity of the track
- v : absolute value of the track velocity

As can be seen from equations 1 and 4, $\lim_{v \rightarrow \infty} \delta = 0 \implies \lim_{v \rightarrow \infty} F_y = 0$. EM levitation is thus more efficient at high speeds. Note that due to the extremely low air pressure ($< 1000 Pa$) the air resistance is also insignificant. The derivation of these equations and the theory behind them are discussed in detail in [3].

2.2 Electrodynamic Wheels

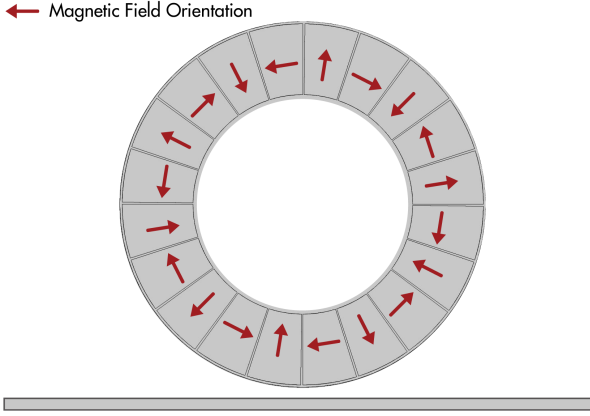


Figure 2: Halbach Wheel Magnetic Field Orientation

EDWs are composed of a circular array of magnets arranged in a Halbach configuration [Figure 2]. As the wheel is spun above or besides a conductive track, thrust and lateral forces are generated in a similar manner to that of a linear array. The analytic form of the equations describing the forces generated is given below.

$$F_x = \frac{1}{4\pi\mu_0} Re \left[\int_{-\infty}^{\infty} B_x^* B_y d\xi \right] \quad (5)$$

$$F_y = \frac{1}{8\pi\mu_0} Re \left[\int_{-\infty}^{\infty} (B_y^* B_y - B_x^* B_x) d\xi \right] \quad (6)$$

With:

$$B_y(\xi, y) = -j\xi T(\xi, y) * B_{xy}^s(\xi, b) \quad (7)$$

$$B_x(\xi, y) = \frac{\partial T(\xi, y)}{\partial y} B_{xy}^s(\xi, b) \quad (8)$$

The term T has many other dependencies on the geometrical properties of the wheel and the strength of the magnets, but in order to keep this section concise and not go through the full derivation of these equations, these will not be explained. A detailed derivation of these equations is given by *Nirmal et al.* [2]. The sign of the thrust force is determined by the slip. The slip is defined as the relative velocity between the track and the point of the wheel lying directly above it:

$$s = \omega r_o - v \quad (9)$$

Where ω and r_o are the rotational velocity and the outer radius of the wheel respectively, and v is the translational

velocity of the wheel. Thus a positive slip will produce a positive force (thrust) and a negative slip will produce a negative force (brake) [Figure 3].

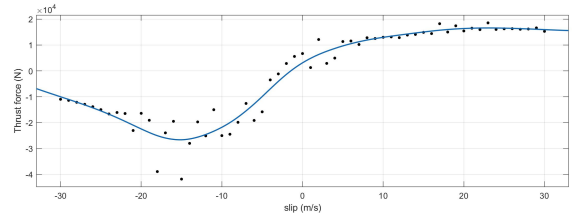


Figure 3: EDW thrust force against slip

3 Simulation Methods

Different COMSOL modules were used to simulate the fields and forces produced by the systems described in the previous section. A numerical model of the equations presented in the previous section was also created in the MATLAB programming language.

3.1 COMSOL

3.1.1 Halbach Levitation Arrays

The Rotating Machinery, Magnetic interface was used to simulate a linear magnetic array moving on top of a conductive aluminium track in 2D. A stationary study was conducted with the following boundaries:

- Continuity between the boundary of the array and the air
- Continuity between the boundary of the track and the air
- Magnetic Insulation boundary in the air surrounding the system.

These boundaries are shown graphically on Figure 4. The study was parametrised according to the velocity of the array v , taking values from 0 to 75m/s.

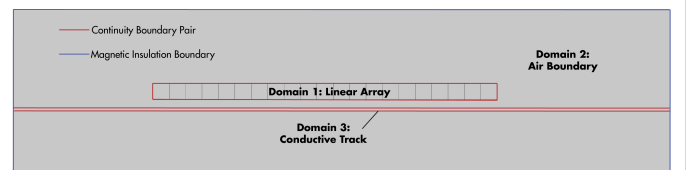


Figure 4: Halbach Array COMSOL Model Boundaries

3.1.2 Electrodynamic Wheels

The Rotating Machinery, Magnetic interface was used to simulate an EDW spinning on top of a conductive aluminium track in 2D. A time dependant study was conducted with the following boundaries:

- Continuity between the boundary of the wheel and the air
- Continuity between the boundary of the track and the air
- Magnetic Insulation boundary in the air surrounding the system.

These boundaries are shown graphically in Figure 5. The study was parametrised according to s and v , the slip and the translational velocity of the wheel. Thus, the frequency of the rotating domain was determined by using equation 9 and solving for ω . The time dependant solver was configured with the following settings:

- Times: 0 – 50ms
- Timestep: 0.5ms
- Relative Tolerance: 0.001

Also, the maximum safety was ensured by performing a study in the worst case scenario when the pod stops using secondary (emergency) braking system but with EDWs still spinning above (besides) the track. It was estimated that it would take about 3s to decelerate from 6000RPM to rest. Therefore, for this special case Heat Transfer in Solids and Rotating Machinery coupled interfaces in Multiphysics branch were used. It was assumed that the general heating occurs due to Joule’s First Law and the heat is dissipated in accordance with Volumetric Loss Density function.

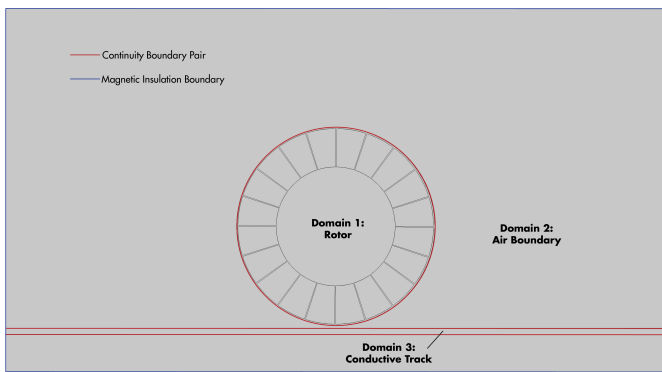


Figure 5: Electrodynamic Wheel COMSOL model boundaries

3.2 MATLAB Numerical Model

A numerical model implementing the equations presented in the previous section was developed in the MATLAB programming language. The integration over Fourier space was

performed numerically using the *quadgk* method [4], which used the Gauss-Kronrod quadrature. This model was used to simulate the pod’s trajectory using an Euler step but also to check the results from the COMSOL simulations.

3.3 Model parameters

The following parameters were chosen for each of the systems. The objective was to validate whether this design would provide the necessary thrust (EDWs) and lift forces (Levitation arrays). The EDWs and Halbach arrays were configured using the parameters represented in Table 1 and Table 2 respectively.

Parameters	Value
Outer radius	0.1428m
Inner radius	0.092m
Number of pole pairs	5
Remnant magnetic field of N52 sector magnets	1.42T

Table 1: Halbach wheel parameters

Parameters	Value
Edge of cubic magnet	25mm
Number of magnets per ski	21
Remnant field of each magnet	1.42T
Thickness of the track	12.7mm
Conductivity of the track	$2.92 \times 10^{-8} \Omega m$

Table 2: Halbach array parameters

4 Results and Discussion

In this section, the magnetic fields and forces generated by EM levitation and propulsion systems are presented as well as the thermal map of the track heating due to eddy currents.

4.1 Fields

The magnetic fields produced by the linear Halbach array and EDW can be seen in Figures 6 and 7 respectively. As expected, the field is sinusoidal. Maximum magnetic flux density near the surface of Halbach wheel is ≈ 0.7 T.

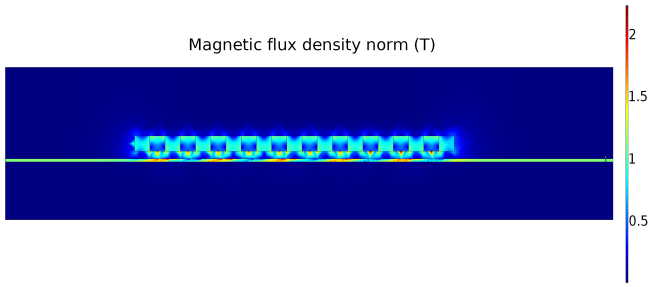


Figure 6: Magnetic field generated by linear Halbach array in 2D.

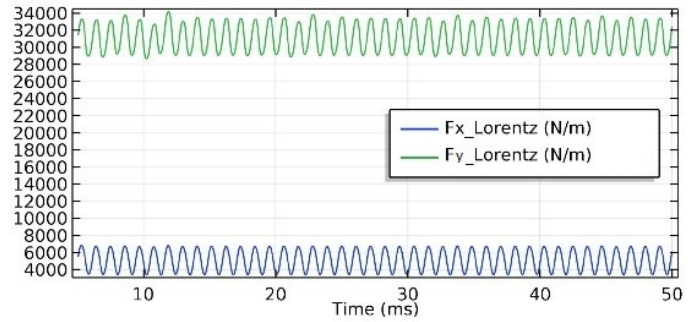


Figure 8: EDW thrust force against time for 10m/s slip and 30m/s translation velocity of the track

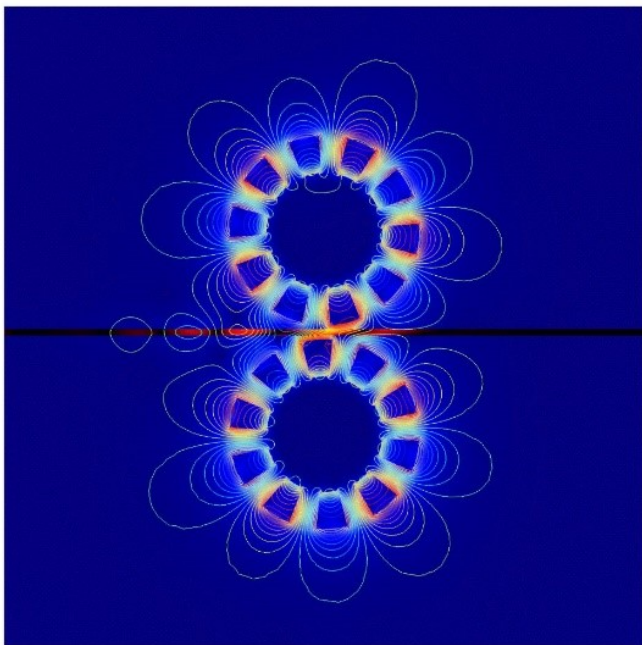


Figure 7: Magnetic field generated by two EDWs in 2D.

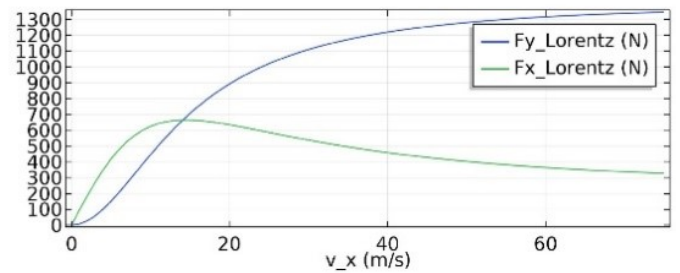


Figure 9: Halbach array forces against velocity.

4.2 Forces

The forces generated by the EDWs are shown in figures 8. In Figure 8 the thrust and lateral forces as a function of time are shown. As can be seen, there is a slight oscillation in the thrust force. This is due to the small gap between the sector magnets, which means that the field created is not properly sinusoidal.

The forces generated by the Halbach array as function of velocity can be seen in Figure 9. As is expected, y component of the Force approaches a constant value equal to the weight of the pod, as the velocity increases, and the x component asymptotically approaches 0.

4.3 Heat

The thermal map, obtained by performing the heat transfer simulation of spinning EDW besides the track for 3s, can be seen in Figure 10. The maximum temperature of the track reaches 600K which is what one could expect, as the air gap $g = 5mm$ between the wheel and the guideway is quite small.

Time=3 s Surface: Temperature (K)

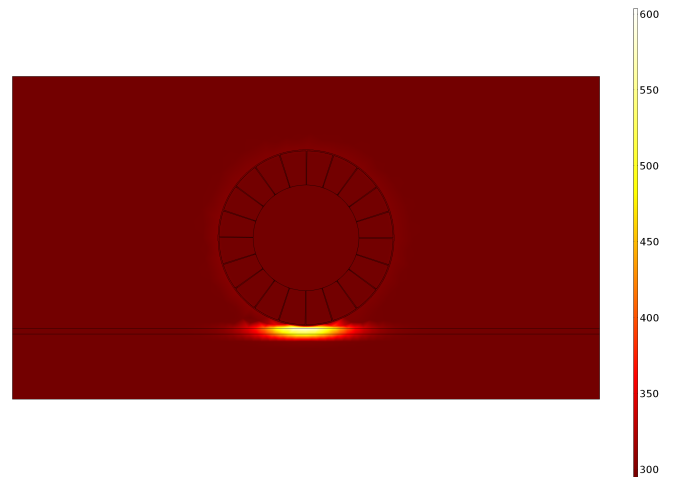


Figure 10: Temperature profile for the rail after 3s of EDW spinning above it.

5 Conclusion

Simulations were run in COMSOL to determine and confirm the design parameters of HYPED's prototype pod levitation and propulsion systems. It was estimated that each EDW provides $\approx 1300\text{N}$ thrust force which should be enough to reach a maximum longitudinal acceleration of 9.81 m/s^2 . The maximum velocity the pod would be able to achieve is about 70m/s .

In the worst case scenario, when the pod is stopped with EDWs still spinning besides the track, the rail would heat up to 600K in 3s while the EDWs are decelerating. Therefore, the heating would be within normal range and would not cause any structural damage to the track.

6 Acknowledgements

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References

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