Poroelasticity Benchmarking for FEM on Analytical Solutions

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Introduction: We examine the poroelastics mode, which couples hydraulics and mechanics by some basic benchmarks. For cases with analytical solutions we check the accuracy for changing meshes and calculate the convergence rate.

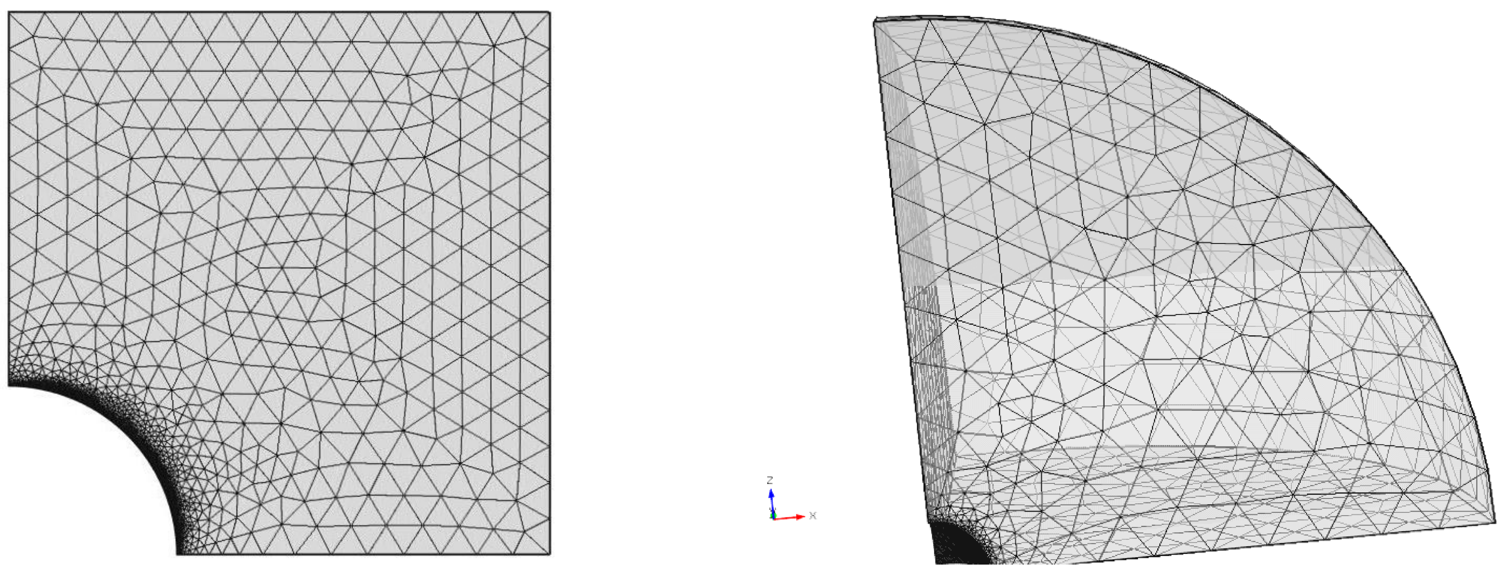


Figure 1. Meshes for 2D and 3D models

Computational Methods: In poroelastics hydraulic and mechanical processes are coupled, which allows the simultaneous modeling of these processes (HM-coupling) in porous media. Applications for this type of modeling are generally necessary in all situations, in which flow processes through a porous material are accompanied by deformations. The pore pressure as a variable is responsible for fluid flow and movements of the solid, and also depends on both fluid and solid states. The coupled set of equations is:

$$-\nabla\cdot\boldsymbol{\sigma} = \mathbf{F}_V, \boldsymbol{\sigma} = \mathbf{s}$$

$$\mathbf{s}-\mathbf{s}_0 = \mathbf{C}:\left(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}_0-\boldsymbol{\varepsilon}_{inel}\right)-\alpha p\mathbf{I}$$

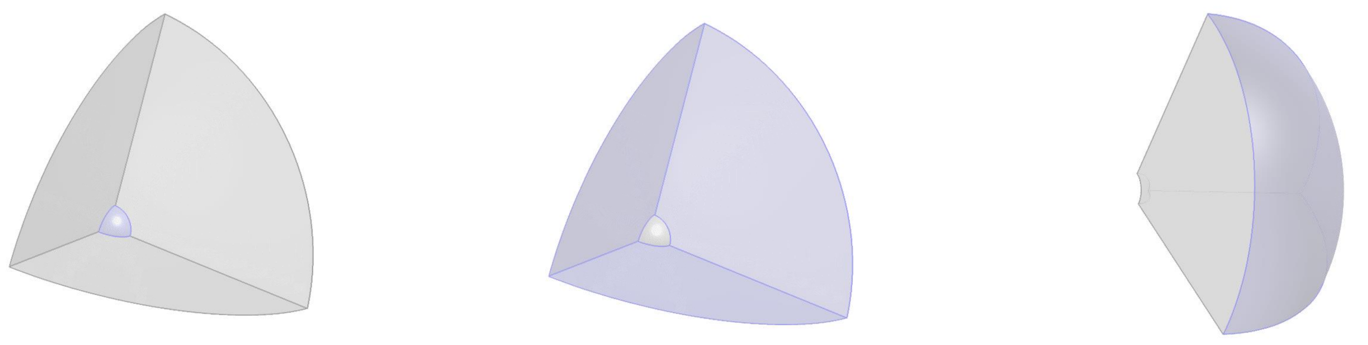
$$\boldsymbol{\varepsilon} = \tfrac{1}{2}\left(\left(\nabla\mathbf{u}\right)^T+\nabla\mathbf{u}\right)$$

$$\rho S\frac{\partial p}{\partial t}+\nabla\cdot\left(\rho\mathbf{q}\right) = Q-\rho\alpha\frac{\partial\varepsilon_V}{\partial t}$$

$$\mathbf{q} = -\frac{k}{\mu}\nabla p$$

Models: Circular cavity (2D) & spherical cavity (3D)

Boundary conditions (for spherical cavity)



free roller load

Table 1. Model parameter list

Variable	Value	Units
Young modulus	52	GPa
Poisson ratio	0.1486	-
Permeability	10^{-17}	m^2
Density	2422	kg/m^3
Porosity	0.08	-
Biot parameter	0.25	-

Results:

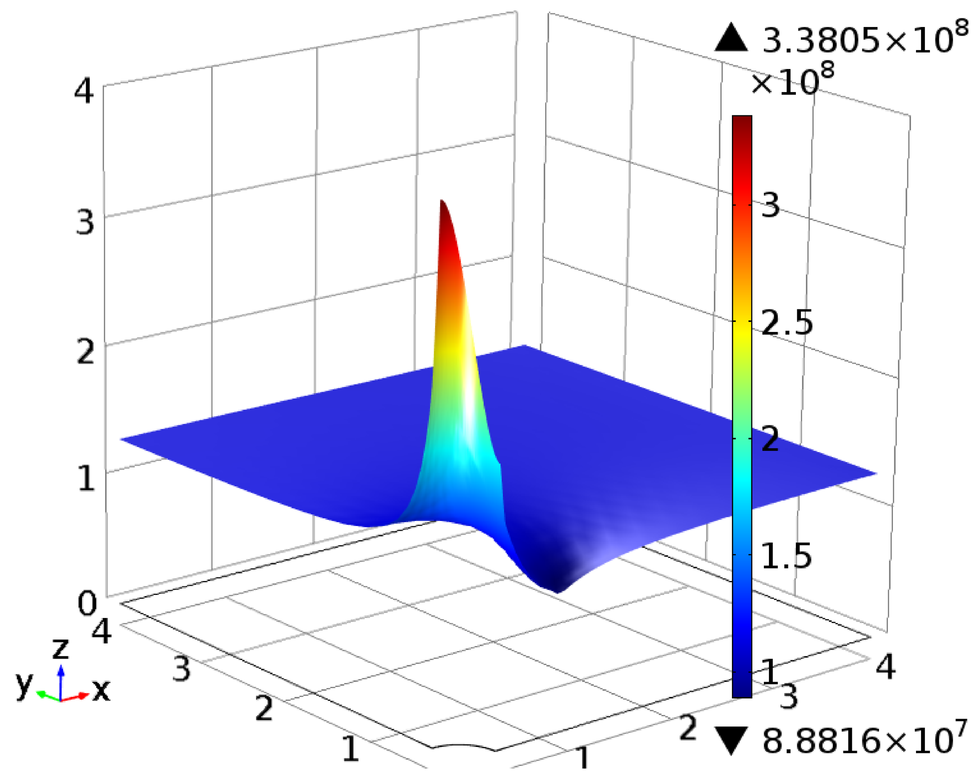


Figure 2. Tresca stress [Pa] of 2D reference problem in deformed mesh for circular cavity

Numerical results are compared with analytical solutions:

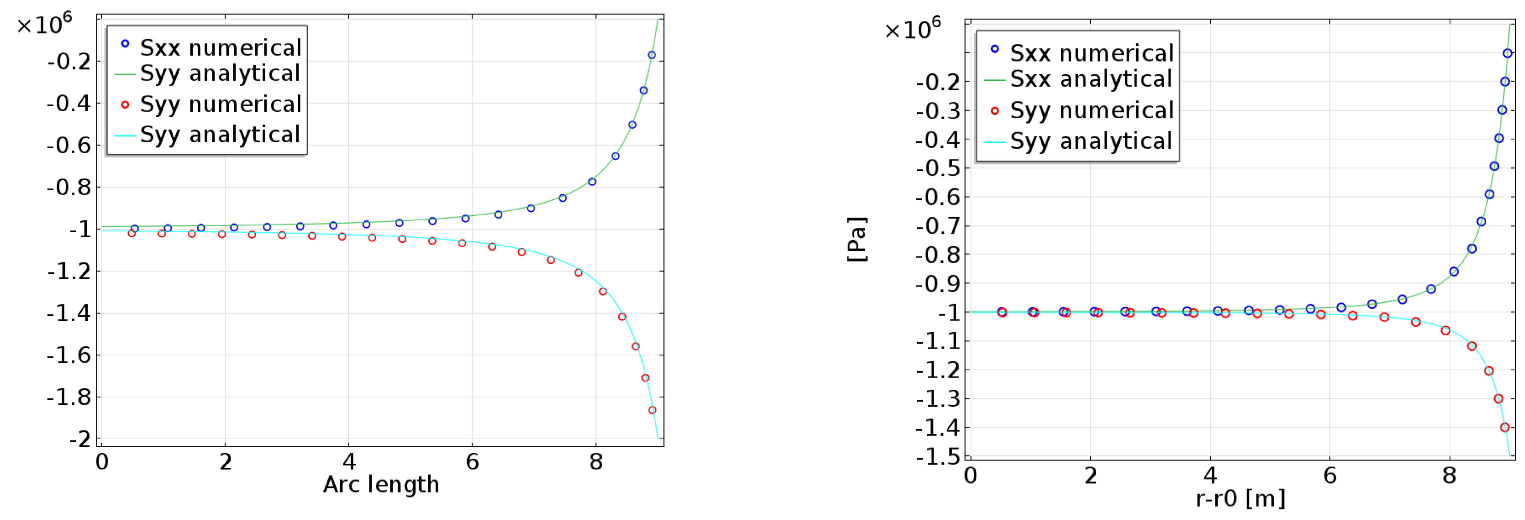


Figure 3. Stress tensor components in dependence of distance from inner radius for circular hole (left: 2D, right: 3D); comparison with analytical solution

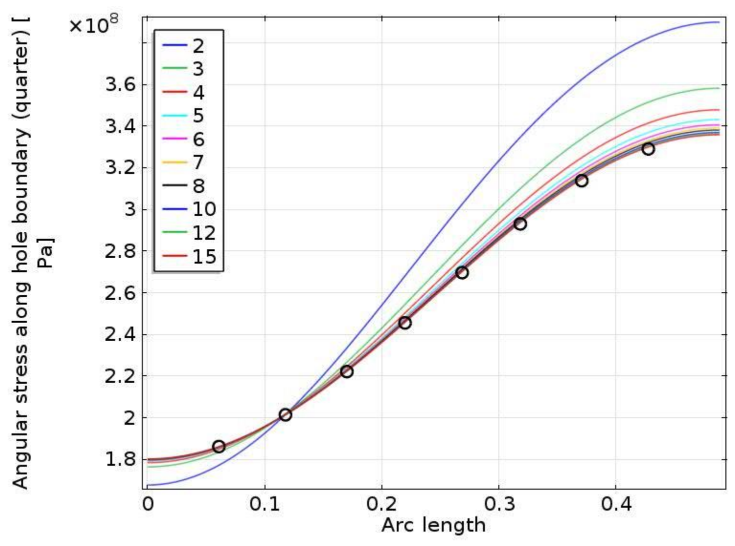


Figure 4. Angular stress along hole boundary in dependence of model extension

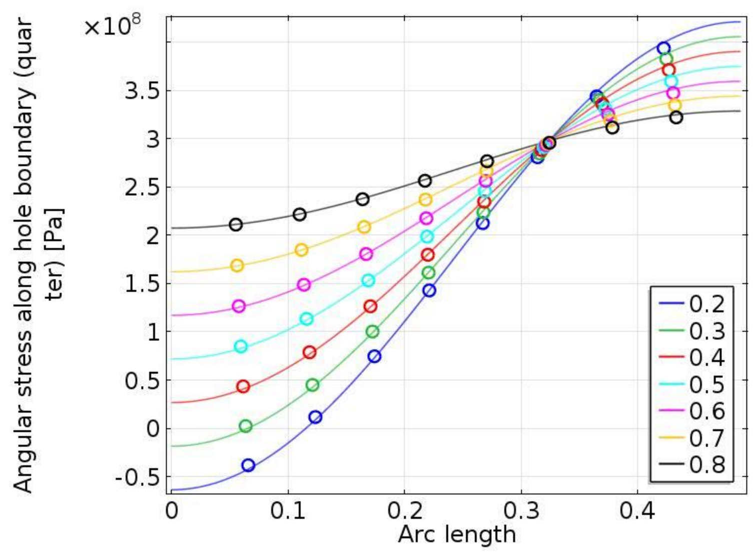


Figure 5. Angular stress along hole boundary in dependence of stress contrast at outer boundaries

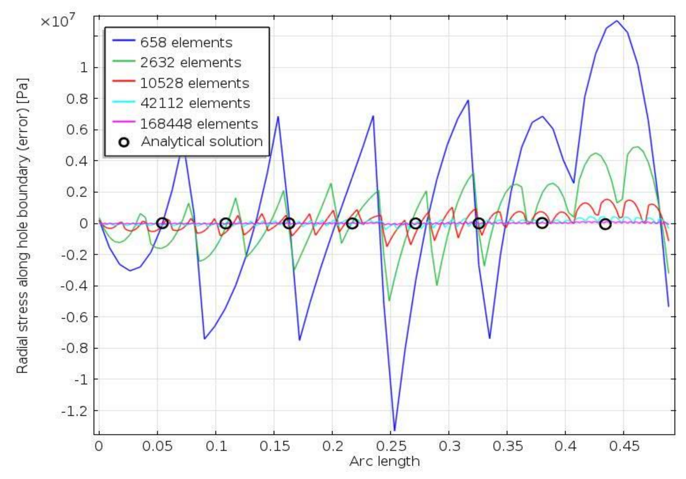


Figure 6. Radial stress error along hole boundary in dependence of grid refinement

Conclusions: Cavity models have been examined in two and three dimensions. Comparisons with analytical solutions show that the finite element models, set up using COMSOL Multiphysics, perform accurately.
With increasing mesh refinement the convergence rate for the 2D model becomes almost quadratic for the 2D cavity. For the 3D cavity it is almost cubic for coarse meshes. However, using the mesh refinements, predefined in COMSOL Multiphysics, the convergence rate decreases to almost quadratic.

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